Indiana Department of Education
SUPPORTING STUDENT SUCCESS

High School Mathematics Pathways: Helping Schools and Districts Make an Informed Decision about High School Mathematics

Defining the Two Pathways
For the purposes of planning for high school curriculum, Indiana Department of Education (IDOE) is setting forth two possible paths for high school mathematics: a traditional pathway and an integrated pathway. It is important to note that *Indiana is not mandating either approach;* however, IDOE encourages districts to carefully consider both options before making a decision.

Rather, the pathways are models of two approaches to organizing content into coherent, comprehensive and rigorous courses that should lead Indiana's students to college and career readiness. Schools are encouraged to read about these pathways in order to determine what best suits their individual needs. Print versions of these course described below, including the standards for each potential course, are available on the Department of Education website.

Traditional Pathway
This option involves offering three high school courses called Algebra I, Geometry, and Algebra II. Although these courses retain their traditional names, the standards for each, and the organization of the standards, are significantly different than the current Indiana courses. In addition, each of the three new courses includes a unit in Probability and Statistics.

Integrated Pathway
The second option involves three high school courses called Integrated Mathematics I, Integrated Mathematics II, and Integrated Mathematics III. The integrated courses include Algebra, Geometry, and Statistics standards in each course. This integration allows students not only to continue to systematically build proficiency in each domain (Algebra, Geometry, and Statistics/Probability) each year, but also attempts to help students see the connections and interrelationships between these three domains of mathematics.

Choosing a Pathway: Common Misconceptions
Before choosing a pathway, school corporations might wish to consider common misconceptions associated with the high school pathways.
Three common misconceptions are:

1) The requirements of integrated mathematics are less rigorous than of traditional mathematics courses;
2) State assessments are only aligned with the traditional pathway; and
3) Integrated textbooks only teach through "applied" situations and fail to address procedural fluency. These misconceptions are described in greater detail below.

Rigor of Requirements

Standards covered in the three-year integrated sequence are the exact same standards as those covered in a three-year traditional sequence. Although in the past, integrated mathematics courses were typically offered only to students who were unable to succeed in a traditional Algebra or Geometry class; this need not be the case. In designing the new model pathways, Achieve and other national mathematics experts paid close attention to *ensuring that both paths are equally rigorous*. The difference between the two courses lies in the sequencing of the standards, rather than in the standards themselves.

Alignment of Assessments

During the 1990s and early 2000s, a number of districts across the state offered integrated courses to all students. Although some of these programs were successful, most were abandoned when the state graduation qualifying exam (GQE) was changed to the current Algebra I End-of-Course Assessment (ECA) model, which tests each Algebra I standard. In order to provide students with an opportunity to learn what would be on this critical exam, most schools switched back to the traditional Algebra courses and a Geometry course. Schools that currently offer integrated courses must alter their testing schedule to accommodate for these differences, usually not requiring students to take the Algebra ECA until midway through Integrated Mathematics II.

Although this model may have worked to a certain extent in the past, its effectiveness will be limited as we move to Common Core assessments. Indiana is a governing member of the Partnership for Assessment of Readiness of College and Careers (PARCC), through which an assessment system will be developed that 1) provides quarterly, instead of yearly, assessments, and 2) extends through Algebra II or Integrated Mathematics III. To accommodate for the differences between the two pathways, beginning in 2014-15 PARCC will have assessments available for both integrated and traditional courses. Schools will retain the ability to choose their own pathway for high school courses and subsequently will be able to choose which set of assessments (integrated or traditional) to assign to their students. The assessments will be equivalent in structure, question type, and rigor, with the only difference being the content expectations for individual assessments. By the end of the 3-year high school assessment schedule, the standards tested for each pathway will be equivalent.

Applications and Procedures

To many high school mathematics teachers, the terms “integrated” and “applied” or “in context” are synonymous. Although many integrated textbooks present material in applied situations, the two terms are not inextricably linked. The current *Pre-Calculus* course, for example, follows an integrated model, incorporating algebra, trigonometry, geometry, and data analysis into one course without ever being viewed as “applied math.” Grades K-8 are also integrated courses.

The strength of the integrated model is not that standards are taught in context, but rather that they are viewed as connected to other standards, both within and across mathematical domains. Understanding and utilizing these connections becomes vital to long-term success in mathematics and STEM disciplines – which has become important for all students, as nearly all career paths now require students to at least be proficient in the STEM disciplines (IDOE STEM guidance document).

In order to appropriately attend to the Standards for Mathematical Practice, both traditional and integrated curricula should attend to both mathematical and real-world situations. Students enrolled in either pathway should arrive at the same point by the end of the third year, and as such teachers in both pathways should ensure that they are building similar competencies.

Comparing the Pathways

The following section provides an in depth analysis of the two pathways, as outlined by Achieve and adopted by the Indiana State Board of Education. In determining which pathway to pursue, corporations must be cognizant not only of the difference between the two pathways, but also the differences between previous Indiana high school courses and the new Common Core courses.

The traditional pathway under the Common Core State Standards is organized in a much different manner than the traditional Algebra I-Geometry-Algebra II courses. The focus on "big ideas" or "critical areas" requires educators to shift their attention from discrete standards to integrated units when designing curriculum, instruction, and assessment, regardless of the path chosen. The connections between standards, both within and across domains, should be emphasized.

First, there is considerable overlap between the arrangements of standards in the courses at the *unit level*. The integrated pathway has 16 units, and the titles for 15 of the units match the titles from the traditional pathway. The last unit, *Extending the Number System*, has no match in the traditional pathway. The same is true for the traditional pathway: 14 of its 15 units are matched in the integrated pathway, with the last unit, *Extending to Three Dimensions*, having no match.

Although the unit titles are the same, the standards that fall into each unit for the separate pathways are somewhat different, with a good deal of overlap. Ten units have at least 75 percent of the standards in common between the two pathways).

Statistics and Probability

The statistics and probability requirements perfectly match between the two pathways, and in both these standards are evenly distributed throughout the three courses (*Descriptive Statistics* in the first course, *Applications in Probability* in the second, and *Inferences and Conclusions from Data* in the third). Because statistics and probability standards have not generally been in the required high school curriculum, it is conceivable that many current teachers will need support in the instruction of these standards, regardless of pathway.

Geometry

The largest difference between the two sequences, not surprisingly, lies in the sequencing of the six geometry-related domains, which primarily appear in five overlapping units: *Congruence Proof, and Constructions; Circles with and Without Coordinates; Connecting Algebra and Geometry through Coordinates*; and two trigonometry-related units (which, due to significant differences in the standards, have different names in each pathway). These units make up the bulk of the traditional Geometry course, as well as one unit of the traditional Algebra II course (*Trigonometric Functions*).

These differences are not accidental. In the integrated model, geometry standards can be found in each of the three courses, many times alongside related standards from modeling or functions. The courses are designed around standards and big ideas that are closely related conceptually and developmentally, instead of the common thread being limited to what appears in geometry. As stated earlier, understanding and utilizing the connections across disciplines, not just within disciplines, is important for long-term success in mathematics. By making these connections explicit, teachers can help to ensure that their students are poised for success beyond high school. Although these connections can be emphasized in the traditional pathway, the sequencing of geometry standards in the integrated pathway allows for these connections to permeate each of the three high school courses.

Trigonometry

The expectations for *Trigonometry* in the first three high school courses are significantly different than in previous Indiana high school courses. Because this level of rigor in trigonometry has not generally been seen in the first three years of the high school curriculum, it is conceivable that many current teachers will need support in the mathematics as well as promising instructional strategies for these standards.

Trigonometry in the traditional sequence is spread between Algebra II (when related to functions) and Geometry (when related to triangles), as well as a significant portion falling in fourth-year courses. The same model is used in the integrated sequence, with the standards being spread between Integrated II, III, and other fourth-year courses. The largest difference between the two pathways is how the standards are grouped. The grouping in the traditional sequence revolves around functions or geometry, whereas the integrated sequence incorporates standards from both areas into units revolving around right triangle trigonometry and general trigonometry. The integrated sequence withholds all advanced trigonometry standards until the third course, whereas these standards are included with in the Geometry course in the traditional sequence.

Mathematical Modeling

A clear focus of the Common Core State Standards in Mathematics is to emphasize the importance of modeling in the K-12 curriculum. Modeling is "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (CCSS p 72). Effective mathematical models may incorporate concepts from a variety of mathematical domains, including statistics, algebra, and geometry, as well as others. As a mathematical practice, modeling should permeate both pathways and bridge the gap between academic and real-world problems.

One key difference between the two pathways is the sequencing of a formal modeling unit. In both pathways, the formal modeling unit appears in the third course (*Modeling with Functions* in Algebra II, and *Mathematical Modeling* in Integrated Mathematics III). The key difference between the two is the inclusion of standards from the *Modeling with Geometry* domain in Integrated Mathematics III, whereas this unit in Algebra II restricts the content to algebraic functions. Because of this, *Modeling with Geometry* standards are placed in the *Similarity, Proof, and Trigonometry* unit in the traditional sequence, and thus are disconnected both from the other content standards in this unit and from the modeling standards as a whole. Schools using the traditional sequence should ensure that these standards are connected to other content, not simply taught in isolation. Although proficiency in mathematical modeling can still be built in the traditional pathway, the sequencing of these standards in the integrated pathway more appropriately support the coherence of this mathematical practice.

Advanced (STEM) Standards
In writing the Common Core mathematics standards, the writing teams delineated certain standards that extend beyond the "college and career readiness" expectation. Although these standards, marked with a (+) symbol, were included as standards that should be included in fourth-year courses, both pathways have included advanced standards to build coherence in the courses. In total, ten advanced standards have been included in each pathway: four are related to statistics, two are related to complex numbers, one is related to geometry, and the remaining three are related to trigonometry. In the both pathways, two advanced statistics standards have been spiraled in the second and third courses. In the integrated pathway, the advanced complex numbers standards have also been spiraled in Integrated Mathematics II and III, whereas these standards only appear in Algebra II for the traditional pathway. The trigonometry and geometry standards are all included in Geometry for the traditional pathway, but they are separated into Integrated Mathematics II and III for the integrated courses.

Although these standards will not be assessed on the statewide tests, the design of the units allows for students to master the advanced content alongside the college-and-career-ready content. By scaffolding certain standards and grouping them more appropriately, the integrated sequence should provide students with a better opportunity to build mastery of these advanced standards during the first three years of high school mathematics courses.

Sequence of the Traditional Pathway
Currently in Indiana, school corporations use one of two sequences for high school courses in the traditional pathway: Algebra I–Geometry–Algebra II or Algebra I–Algebra II–Geometry. Under the new courses, this decision will continue to remain under local control; however, as with the pathway decision, schools should carefully consider the implications of both options before making a decision.

A comparison of the standards in Algebra II and Geometry by the Indiana Department of Education revealed that the two courses could be offered in either order, or even simultaneously. However, the more appropriate sequence appears to be Algebra I – Geometry – Algebra II.

Although high schools have the option to sequence these courses as they deem appropriate, schools should consider the following observations when making this decision.

- The sequencing of the trigonometry standards follows more logically from Geometry to Algebra II than it does from Algebra II to Geometry. The *Trigonometry* standards in Geometry develop a student's understanding of trigonometric ratios using triangles, whereas the *Trigonometry* standards in Algebra II develop a functional understanding of trigonometric ratios. Although either can be taught first, developmentally it may be more appropriate to introduce the Geometry unit before the Algebra II unit.
- A comparison of Geometry and Algebra II to Integrated Mathematics II and III reveals a much closer alignment between Geometry and Integrated Mathematics II than Integrated Mathematics III, and between Algebra II and Integrated Mathematics III than Integrated

Mathematics II. This suggests that the standards in Geometry are more likely to be precursors for Algebra II than vice versa.

- The sequencing of the probability and statistics standards follows a more logical order from Geometry to Algebra II than vice versa. The probability unit in Geometry (Applications of Probability) will strengthen a student's understanding of the data analysis unit in Algebra II (Inferences and Conclusions from Data) more than the data analysis unit would inform the probability unit. In fact, Applications of Probability in many ways could be viewed as a prerequisite for Inferences and Conclusions from Data.
- The transition to fourth-year math courses will likely be easier from Algebra II than from Geometry. Most of the advanced standards that are not included in the first three courses more closely align to Algebra II than Geometry, and as such it may be beneficial for students to take Algebra II immediately before Precalculus or other 4th-year math courses.
- Many districts have moved to the Algebra I – Algebra II – Geometry sequence to provide students with remediation in Algebra I ECA content while they are taking Algebra II. With the PARCC assessments, high school assessments will be administered for all three courses (Algebra I, Geometry, and Algebra II). As such, the benefit of remediation is diminished.
- Both sequences involve a one-year absence of Algebra-related standards for students. To ensure students still retain a working knowledge of Algebra-related content, Geometry teachers should integrate Algebra into their lessons, as appropriate.

Building Foundations for High School Courses
Although the two pathways have nuances that make them distinct, the requisite foundations for success in either pathway are the same. Students must not only have demonstrated proficiency in the K-8 Standards for Mathematical Content, but also in the Standards for Mathematical Practice. Students with weak foundations in Mathematical Content may struggle to explain mathematical procedures, develop connections between related standards, consistently make computations, or retain knowledge from previous grades. Students with weak foundations in Mathematical Practice may struggle to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut.

As corporations consider the proper course of action for high school courses, an analysis of 6-8 or even K-8 programs may also be appropriate. The goal is that students graduate high school as "college and career ready," and this path starts long before students enter high school. Schools should implement policies that allow all students to arrive in high school with the proper foundations to begin their high school course of study with either Algebra I or Integrated Mathematics I (or a higher-level course). To provide support for students who enter high school without these necessary foundations, schools may need to develop support courses that can be offered concurrently with Algebra I or Integrated Mathematics I, allowing students to strengthen weak foundations while continuing their progression towards college and career readiness.