



**Indiana Academic Standards for Mathematics – Calculus
Adopted April 2014 – Standards Resource Guide Document**

This Teacher Resource Guide has been developed to provide supporting materials to help educators successfully implement the Indiana Academic Standards for Calculus Mathematics – Adopted April 2014. These resources are provided to help you in your work to ensure all students meet the rigorous learning expectations set by the Academic Standards. Use of these resources is optional – teachers should decide which resource will work best in their school for their students.

This resource document is a living document and will be frequently updated.

Please send any suggested links and report broken links to:

Bill Reed

Secondary Math Specialist - Indiana Department of Education

wreed@doe.in.gov - 317-232-9114

The Indiana Department of Education would like to thank Linda Streepy for her contributions to this document.

The examples in this document are for illustrative purposes only, to promote a base of clarity and common understanding. Each example illustrates a standard but please note that examples are not intended to limit interpretation or classroom applications of the standards.

The links compiled and posted in this Resource Guide have been provided by the Department of Education and other sources. The DOE has not attempted to evaluate any posted materials. They are offered as samples for your reference only and are not intended to represent the best or only approach to any particular issue. The DOE does not control or guarantee the accuracy, relevance, timeliness, or completeness of information contained on a linked website; does not endorse the views expressed or services offered by the sponsor of a linked website; and cannot authorize the use of copyrighted materials contained in linked websites. Users must request such authorization from the sponsor of the linked website.

GOOD WEBSITES FOR MATHEMATICS:

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.math.hope.edu/swanson/methods/applets.html>

<http://learnzillion.com>

<http://illuminations.nctm.org>

<https://teacher.desmos.com>

<http://illustrativemathematics.org>

<http://www.insidemathematics.org>

<https://www.khanacademy.org/>

<https://www.teachingchannel.org/>

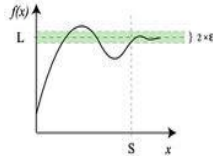
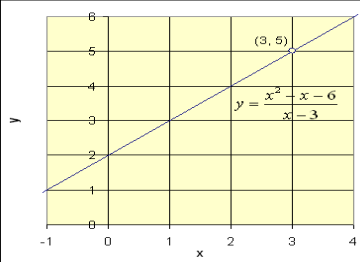
<http://map.mathshell.org/materials/index.php>

<https://www.istemnetwork.org/index.cfm>

<http://www.azed.gov/azccrs/mathstandards/>

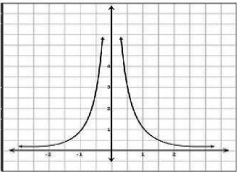
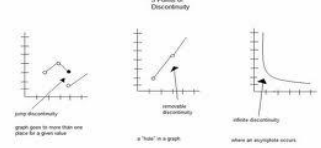
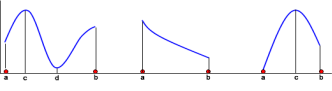


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	Indiana Academic Standards for Mathematics Calculus – Adopted April 2014 – Resource Guide Document	Highlighted Vocabulary Words from the Standard Defined	Specific Calculus Example for the Standard	Specific Calculus Electronic Resource for the Standard
Limits and Continuity				
MA.C.LC.1:	C.LC.1: Understand the concept of limit and estimate limits from graphs and tables of values.	A limit is the function value as the x-value gets arbitrarily close to a single number from both the positive and negative directions.		http://apcalcnotebookarjw.wikispaces.com/Estimating+limits+from+graphs+or+tables+using+data
MA.C.LC.2:	C.LC.2: Find limits by substitution.		$\lim_{x \rightarrow 4} \frac{\frac{1}{4} + \frac{1}{x}}{\frac{4}{4+x}} = \lim_{x \rightarrow 4} \frac{\frac{x+4}{4x} + \frac{4}{4x}}{\frac{4x}{4+x}}$ $= \lim_{x \rightarrow 4} \frac{\frac{x+4}{4x}}{\frac{4x}{4+x}}$ $= \lim_{x \rightarrow 4} \frac{x+4}{4x} \cdot \frac{1}{x+4}$ $= \lim_{x \rightarrow 4} \frac{1}{4x} = \frac{1}{4(-4)} = -\frac{1}{16}$	https://fp.auburn.edu/holmerr/1617/Textbook/limbysubst-screen.pdf
MA.C.LC.3:	C.LC.3: Find limits of sums, differences, products, and quotients.		http://www.youtube.com/watch?v=K2ORnyzTP34&safe=active	http://www.analyzemath.com/calculus/limits/properties.html
MA.C.LC.4:	C.LC.4: Find limits of rational functions that are undefined at a point.			http://www.rose-hulman.edu/media/89584/lclimitsguide.pdf
MA.C.LC.5:	C.LC.5: Find limits at infinity.		$\lim_{x \rightarrow \infty} \frac{x^3 - 2}{5x^4 - 3x^3 + 2x} = \lim_{x \rightarrow \infty} \frac{x^3 \left(1 - \frac{2}{x^3}\right)}{x^4 \left(5 - \frac{3}{x} + \frac{2}{x^3}\right)}$ $= \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right) \left(\frac{1 - \frac{2}{x^3}}{5 - \frac{3}{x} + \frac{2}{x^3}}\right)$ $= (0) \left(\frac{1 - 0}{5 - 0 + 0}\right)$ $= (0)$	http://www.mathsisfun.com/calculus/limits-infinity.html http://tutorial.math.lamar.edu/Classes/CalcI/LimitsAtInfinityI.aspx



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MA.C.LC.6:	C.LC.6: Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. Find special limits $\lim_{x \rightarrow 0} \frac{\sin x}{x}$		 $\lim_{x \rightarrow 0} \frac{1}{x^2}$ <p>Vertical Asymptote – No Limit Ever Reached</p>	http://maretbccalculus2007-2008.pbworks.com/w/page/20301392/Describing%20asymptotic%20behavior%20in%20terms%20of%20limits%20involving%20infinity
MA.C.LC.8:	C.LC.8: Understand continuity in terms of limits.	A function f is continuous at c if the following conditions are met: $f(c)$ is defined, the limit of $f(x)$ as x approaches c exists, and the limit of $f(x)$ equals $f(c)$ as x approaches c .	http://www.youtube.com/watch?v=SBzaRMao7rs&safe=active	https://centralmatteacher.wikispaces.com/Understanding+continuity+in+terms+of+limits
MA.C.LC.9:	C.LC.9: Decide if a function is continuous at a point.			http://www.math.psu.edu/math110/ic7.pdf
MA.C.LC.10:	C.LC.10: Find the types of discontinuities of a function.		 <p>3 Types of Discontinuity</p> <p>Jump discontinuity: a function with a jump at a given value. Point discontinuity: a hole in a graph. Infinite discontinuity: where an asymptote occurs.</p>	http://archives.math.utk.edu/visual.calculus/1/continuous.5/ http://www.math.brown.edu/utra/discontinuities.html
MA.C.LC.11:	C.LC.11: Understand and use the Intermediate Value Theorem on a function over a closed interval .	Intermediate Value Theorem: If f is continuous on the closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number, c , in $[a, b]$ such that $f(c) = k$. A closed interval is an interval that includes all of its limit points. If the endpoints of the interval are finite numbers a and b , then the interval $\{x: a \leq x \leq b\}$ is denoted $[a, b]$. If one of the endpoints is \pm infinity, then the interval still contains all of its limit points (although not all of its endpoints), so $[a, \infty)$ and $(-\infty, b]$ are also closed intervals, as is the interval $(-\infty, \infty)$.	http://www.youtube.com/watch?v=6AFT1wnid9U&safe=active	http://www.mathsisfun.com/algebra/intermediate-value-theorem.html
MA.C.LC.12:	C.LC.12: Understand and apply the Extreme Value Theorem : If $f(x)$ is continuous over a closed interval, then f has a maximum and a minimum on the interval.	If f is continuous on a closed interval $[a, b]$, then f has both a minimum and maximum on the interval. Relative minimum is the least possible value of f over an open interval. Relative maximum is the greatest possible value of f over an open interval. The absolute minimum is the least possible value on the entire function f . The absolute maximum is the greatest possible value on the entire function f .	<p>Extreme Value Theorem: If f is continuous over a closed interval, then f has a maximum and minimum value over that interval.</p>  <p>Identify where the absolute minimum and maximum values are located in each of the three cases above.</p>	http://oregonstate.edu/instruct/mth251/cq/Stage4/Lesson/EVT.html



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Differentiation				
MA.C.D.1:	C.D.1: Understand the concept of derivative geometrically , numerically , and analytically , and interpret the derivative as a rate of change.	A derivative $f'(x)$ can be found geometrically by calculating the slope of line tangent to $f(x)$ at x . A limit can be estimated numerically by constructing a table of values or it can be calculated analytically by using algebra or calculus.		http://maretbccalculus2007-2008.pbworks.com/w/page/20301389/Derivative%20presented%20geometrically%2C%20numerically%2C%20and%20analytically
MA.C.D.2:	C.D.2: State, understand, and apply the definition of derivative.		http://www.youtube.com/watch?v=vzDYOHETFlo&safe=active	http://www.sosmath.com/calculus/diff/der00/der00.html
MA.C.D.3:	C.D.3: Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions.		$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$ $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x.$	http://www.math.brown.edu/utra/trigderivs.html http://www.intmath.com/differentiation-transcendental/7-applications-derivatives-log-exponential.php
MA.C.D.4:	C.D.4: Find the derivatives of sums, products, and quotients.		Examples: $\frac{d}{dx}[x^2 + 2x] = \frac{d}{dx}[x^2] + \frac{d}{dx}[2x] = 2x + 2$ $\frac{d}{dx}[x^2 - 2x] = \frac{d}{dx}[x^2] - \frac{d}{dx}[2x] = 2x - 2$	http://www.suluclac.com/Derivative+rules+for+sums+products+and+quotients+of+functions
MA.C.D.5:	C.D.5: Find the derivatives of composite functions, using the chain rule .	The chain rule is a formula for computing the derivative of the composition of two or more functions		https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/chainruledirectory/ChainRule.html http://faculty.atu.edu/mfinan/2243/business33.pdf
MA.C.D.6:	C.D.6: Find the derivatives of implicitly-defined functions .	Differentiation is taking place with respect to x for every variable, which means that when you differentiate a term involving y , you must apply the Chain Rule.	Example: Implicit differentiation is used to find $\frac{dy}{dx}$ for $x^2 + xy - y^2 = 1$. $\frac{d}{dx}(x^2 + xy - y^2) = \frac{d}{dx}(1)$ $2x + (1 \cdot y + x \frac{dy}{dx}) - 2y \frac{dy}{dx} = 0$ $2x + y = 2y \frac{dy}{dx} - x \frac{dy}{dx}$ $2x + y = \frac{dy}{dx}(2y - x)$ $\frac{2x + y}{2y - x} = \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{2x + y}{-x + 2y} \quad \text{or} \quad \frac{2x + y}{2y - x}$	https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/implicitdiffdirectory/ImplicitDiff.html

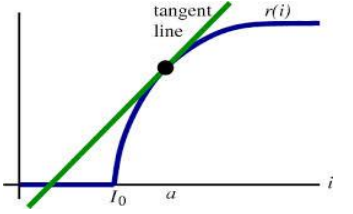
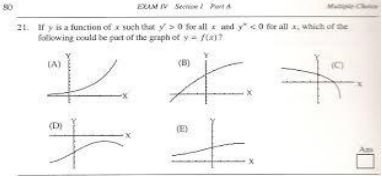
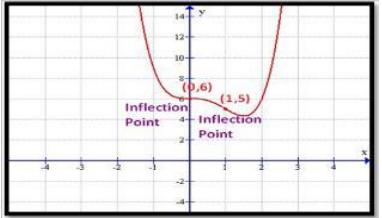
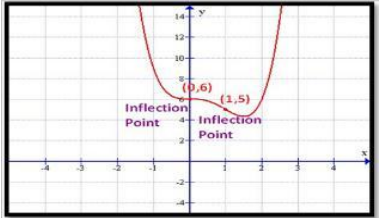


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MA.C.D.7:	C.D.7: Find the derivatives of inverse functions .	A function g is the inverse function of f if: $f(g(x)) = x$ for each x in the domain of g and $g(f(x)) = x$ for each x in the domain of f .	<p>Figure 6.5-1</p>	http://oregonstate.edu/instruct/mth251/cq/Stage6/Lesson/inverseDeriv.html
MA.C.D.8:	C.D.8: Find second derivatives and derivatives of higher order.			http://www.whitman.edu/mathematics/calculus_late_online/section16.06.html http://www.cliffsnotes.com/math/calculus/calculus/the-derivative/higher-order-derivatives
MA.C.D.9:	C.D.9: Find derivatives using logarithmic differentiation .	Using logarithmic properties to simplify differentiation involving products, quotients and power of the second derivative is positive in a given interval, then the graph in that interval is concave up. If the second derivative is negative in a given interval, then the graph in that interval is concave down.	http://www.youtube.com/watch?v=Q27MGf1V70&safe=active	https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/logdiffdirectory/LogDiff.html
MA.C.D.10:	C.D.10: Understand and apply the relationship between differentiability and continuity .	Differentiability implies continuity, continuity doesn't imply differentiability.		http://maretbccalculus2007-2008.pbworks.com/w/page/20301439/Relationship%20between%20differentiability%20and%20continuity
MA.C.D.11:	C.D.11: Understand and apply the Mean Value Theorem .	If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , there exists a number c such that $f'(c) = [f(b) - f(a)] / (b - a)$.	http://www.youtube.com/watch?v=xYOrYlQ3fE0&safe=active	https://www.khanacademy.org/math/differential-calculus/derivative-applications/mean-value-theorem/v/mean-value-theorem



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Applications of Derivatives				
MA.C.AD.1:	C.AD.1: Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents .	A line tangent to a graph that is vertical and has no slope is a vertical tangent. No tangents implies the function is not differentiable at that point		http://maretbccalculus2007-2008.pbworks.com/w/page/20301453/Slope%20of%20a%20curve%20at%20a%20point,%20including%20points%20at%20which%20there%20are%20vertical%20tangents%20and%20points%20at%20whi
MA.C.AD.2:	C.AD.2: Find a tangent line to a curve at a point and a local linear approximation .	Local linear approximation is using the tangent line at a point to approximate relative points.		https://www.khanacademy.org/math/differential-calculus/taking-derivatives/product_rule/v/equation-of-a-tangent-line http://www.math.brown.edu/utra/tangentline.html
MA.C.AD.3:	C.AD.3: Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f' .	If f' is negative, the slope of f is decreasing and if f' is positive, the slope of f is increasing.		http://maretbccalculus2007-2008.pbworks.com/w/page/20301445/Relationship%20between%20the%20increasing%20and%20decreasing%20behavior%20of%20f%20and%20the%20sign%20of%20f%27
MA.C.AD.4:	C.AD.4: Solve real-world and other mathematical problems finding local and absolute maximum and minimum points with and without technology.		http://www.youtube.com/watch?v=votVWz-wKel&safe=active	https://www.math.ucdavis.edu/~jhaddock/Nov21DefinitionsInMyWords.pdf
MA.C.AD.5:	C.AD.5: Analyze real-world problems modeled by curves, including the notions of monotonicity and concavity with and without technology.	Monotonicity means either always increasing or always decreasing. Let f be differentiable on an open interval. The graph of f is concave upward on the interval if f' is increasing on the interval and concave downward on the interval if f' is decreasing on the interval.		http://www.sosmath.com/calculus/diff/der15/der15.html
MA.C.AD.6:	C.AD.6: Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f'' . Understand points of inflection as places where concavity changes.			http://www.sosmath.com/calculus/diff/der15/der15.html

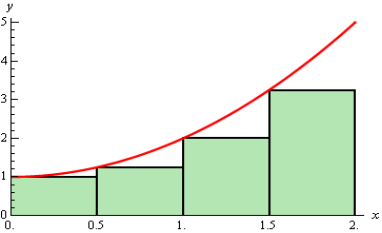
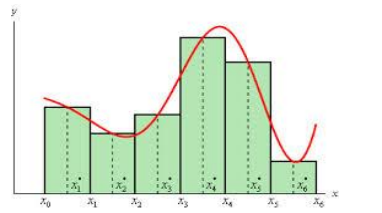


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MA.C.AD.7:	C.AD.7: Use first and second derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology. Compare the corresponding characteristics of the graphs of f , f' , and f'' .		<p>sign of $f'(x)$ direction of $f(x)$</p> <p>sign of $f''(x)$ concavity $f(x)$</p> <p>inc conc down dec conc down dec conc up inc conc up</p>	https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/graphingdirectory/Graphing.html
MA.C.AD.8:	C.AD.8: Use implicit differentiation to find the derivative of an inverse function.			http://www.math.dartmouth.edu/~m3w12/notes/Lecture12-print.pdf https://centralmathteacher.wikispaces.com/Use-of-implicit-differentiation-to-find-the-derivative-of-an-inverse-function
MA.C.AD.9:	C.AD.9: Solve optimization real-world problems with and without technology.	Optimization is an application of calculus involving the determination of the maximum or minimum values using a primary equation and a secondary equation.	<p>Example A rectangle is inscribed in the region in the first quadrant bounded by the coordinate axes and the parabola $y = 1 - x^2$. Find the dimensions of the rectangle that maximize its area.</p>	http://mathitude.perso.sfr.fr/PDF/optimisation%20full%20less%20on.pdf https://www.math.drexel.edu/~jwd25/CALC1_SPRING_06/lectures/lecture9.html
MA.C.AD.10:	C.AD.10: Find average and instantaneous rates of change . Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including distance, velocity, and acceleration.	The average rate of change of a function is defined as the change in x divided by the change in y . The instantaneous rate of change is the slope at one point on a curve.		http://facultypages.morris.umn.edu/~mcquarrb/teachingarchive/Precalculus/Lectures/AverageRateofChange.pdf http://www.math.sc.edu/~diestelr/2.1Notes.pdf
MA.C.AD.11:	C.AD.11: Find the velocity and acceleration of a particle moving in a straight line.		$a = \frac{\Delta v}{\Delta t}$ <p>where $\Delta v = v_f - v_i$ and $\Delta t = t_f - t_i$;</p> <p>therefore, the equation for acceleration may be written as:</p> $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$	http://www.unf.edu/~aschonni/classes/dynamics/HW%20Solutions%20u%2004/Dynamics_CH12_HW.pdf http://www.phy.olemiss.edu/~hamed/Test_Bank_2.pdf
MA.C.AD.12:	C.AD.12: Model rates of change, including related rates problems.	To find the rate of change of two or more related variables that are changing with respect to time.	<p>RATE OF CHANGE MODEL</p> <p>complexity \uparrow rate of change \uparrow</p> <p>ability of organizations to respond \uparrow</p> <p>COPYRIGHT © 2002 MG TAYLOR CORPORATION</p>	http://www.math.dartmouth.edu/~klbooksite/2.01/201.html http://centralmathteacher.wikispaces.com/Modeling+rates-of-change,+including+related+rates+problems



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Integrals				
MA.C.I.1:	C.I.1: Use rectangle approximations to find approximate values of integrals .	Integrals are used to find the area of a region. An approximation can be made by using the sum of the areas of the rectangle(s) contained within function and the x- or y- axis.		https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/riemann-sums/v/simple-riemann-approximation-using-rectangles
MA.C.I.2:	C.I.2: Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points.	A Riemann Sum is a method for approximating the total area underneath a curve on a graph, otherwise known as an integral.		http://math.arizona.edu/~calc/Text/Section7.5.pdf http://www.quia.com/files/quia/users/tcsyoung/calc.--Riemann-sums-packet.pdf
MA.C.I.3:	C.I.3: Interpret a definite integral as a limit of Riemann Sums.		$\text{Riemann sum} = \sum_{i=1}^4 f(c_i)\Delta x_i = \sum_{i=1}^4 [(i - 0.5)^3 + 1] 1$ $= \sum_{i=1}^4 (i - 0.5)^3 + 1$ <p>Enter $\sum ((1 - 0.5)^3 + 1, i, 1, 4) = 66$. The Riemann sum is 66.</p>	http://www.math.wvu.edu/~hjlai/Teaching/Tip-Pdf/Tip1-29.pdf
MA.C.I.4:	C.I.4: Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $\int_a^b f'(x)dx = f(b) - f(a)$		http://www.youtube.com/watch?v=PGmVvigiZx8&safe=active	https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/fundamental-theorem-of-calculus/v/fundamental-theorem-of-calculus http://www.sosmath.com/calculus/integ/integ03/integ03.html



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MA.C.I.5:	C.I.5: Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.		Evaluate $\int_0^1 x(x^2 - 1)^7 dx$. Begin by evaluating the indefinite integral $\int x(x^2 - 1)^7 dx$. Let $u = x^2 - 1$; $du = 2x dx$ or $\frac{du}{2} = x dx$. Rewrite: $\int \frac{u^7 du}{2} = \frac{1}{2} \int u^7 du = \frac{1}{2} \left(\frac{u^8}{8} \right) + C = \frac{u^8}{16} + C = \frac{(x^2 - 1)^8}{16} + C$. Thus the definite integral $\int_0^1 x(x^2 - 1)^7 dx = \frac{(x^2 - 1)^8}{16} \Big _0^1$ $= \frac{(2^2 - 1)^8}{16} - \frac{(0^2 - 1)^8}{16} = \frac{3^8}{16} - \frac{(-1)^8}{16} = \frac{3^8 - 1}{16} = 410$.	http://www.math.brown.edu/~ck9/M0100_Fa11/review_integrals.pdf																
MA.C.I.6:	C.I.6: Understand and use these properties of definite integrals. $\int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx$ $\int_a^b k f(x) dx = k \int_a^b f(x) dx$ $\int_a^a f(x) dx = 0$ $\int_a^b f(x) dx = - \int_b^a f(x) dx$ $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$ If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x) dx \leq \int_a^b g(x) dx$		Properties of definite integrals: If $f(x) \geq 0$ on the interval $[a, b]$, then $\int_a^b f(x) dx \geq 0$ $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ $\int_a^b f(x) dx = - \int_b^a f(x) dx$ $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx, a < c < b$ $\int_a^b f(x) dx = \int_a^c f(a-x) dx$ or $\int_a^b f(x) dx = \int_a^b f(a-b+x) dx$ $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ if $f(-x) = f(x)$ 0 if $f(-x) = -f(x)$	http://www.mathsisfun.com/calculus/integration-definite.html																
MA.C.I.7:	C.I.7: Understand and use integration by substitution (or change of variable) to find values of integrals.	Integration by substitution allows changing the basic variable of an integrand (usually x at the start) to another variable (usually u or v). The relationship between the 2 variables must be specified, such as $u = 9 - x^2$. The hope is that by changing the variable of an integrand, the value of the integral will be easier to determine.	<table border="0"> <tr> <td><i>Expression</i></td> <td><i>Substitution</i></td> </tr> <tr> <td>$(ax + b)^n$</td> <td>$u = ax + b$</td> </tr> <tr> <td>$\sqrt{ax + b}$</td> <td>$u^n = ax + b$</td> </tr> <tr> <td>$a - bx^2$</td> <td>$x = \sqrt{\frac{a}{b}} \sin u$</td> </tr> <tr> <td>$a + bx^2$</td> <td>$x = \sqrt{\frac{a}{b}} \tan u$</td> </tr> <tr> <td>$bx^2 - a$</td> <td>$x = \sqrt{\frac{a}{b}} \sec u$</td> </tr> <tr> <td>$e^x$</td> <td>$u = e^x$</td> </tr> <tr> <td>$\ln(ax + b)$</td> <td>$ax + b = e^u$</td> </tr> </table>	<i>Expression</i>	<i>Substitution</i>	$(ax + b)^n$	$u = ax + b$	$\sqrt{ax + b}$	$u^n = ax + b$	$a - bx^2$	$x = \sqrt{\frac{a}{b}} \sin u$	$a + bx^2$	$x = \sqrt{\frac{a}{b}} \tan u$	$bx^2 - a$	$x = \sqrt{\frac{a}{b}} \sec u$	e^x	$u = e^x$	$\ln(ax + b)$	$ax + b = e^u$	https://www.math.ucdavis.edu/~kouba/CalcTwoDIRECTORY/usubdirectory/USubstitution.html http://tutorial.math.lamar.edu/Classes/CalcI/SubstitutionRuleDefinite.aspx
<i>Expression</i>	<i>Substitution</i>																			
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$\ln(ax + b)$	$ax + b = e^u$																			
MA.C.I.8:	C.I.8: Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.			https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/riemann-sums/v/trapezoidal-approximation-of-area-under-curve http://www.roberson.k12.nc.us/Page/34523																



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Applications of Integrals				
MA.C.AI.1:	C.AI.1: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.	An antiderivative is an integral.		http://centralmathteacher.wikispaces.com/Finding+specific+antiderivatives+using+initial+conditions,+including+applications+to+motion+along+a+line
MA.C.AI.2:	C.AI.2: Solve separable differential equations and use them in modeling real-world problems with and without technology.		<p>Let $u = x^2$; $du = 2x dx$ or $\frac{du}{2} = x dx$.</p> $\int x \sin(x^2) dx = \int \sin u \left(\frac{du}{2} \right) = \frac{1}{2} \int \sin u du = -\frac{1}{2} \cos u + C$ $= -\frac{1}{2} \cos(x^2) + C$ <p>Thus, $y = -\frac{1}{2} \cos(x^2) + C$.</p>	https://www.khanacademy.org/math/differential-equations/first-order-differential-equations/separable-equations/v/separable-differential-equations
MA.C.AI.3:	C.AI.3: Solve differential equations of the form $y' = ky$ as applied to growth and decay problems.			http://www.whitman.edu/mathematics/multivariable/multivariable_17_Differential_Equations.pdf
MA.C.AI.4:	C.AI.4: Use definite integrals to find the area between a curve and the x-axis, or between two curves.		<p style="text-align: center;">Figure 12.3-8</p>	https://www.khanacademy.org/math/integral-calculus/indefinite-definite-integrals/definite-integrals/v/area-between-curves
MA.C.AI.5:	C.AI.5: Use definite integrals to find the average value of a function over a closed interval.	$\frac{1}{b-a} \int_a^b f(x) dx$		http://www.cs.nccu.edu/~melikyan/mat_ns/lec/lec14.ppt
MA.C.AI.6:	C.AI.6: Use definite integrals to find the volume of a solid with known cross-sectional area.			https://www.khanacademy.org/math/integral-calculus/solid-revolution-topic http://www.rit.edu/~wasc/documents/services/resources/handouts/Volumes_by_integration1.pdf
MA.C.AI.7:	C.AI.7: Apply integration to model and solve (with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.			http://staff.chardon.k12.oh.us/webpages/data/sbrown/files/C_hap_7_book.pdf