Avon High School **Name** (Key)

AP Calculus AB

UNIT 4 EXAM REVIEW

Sections 3.1-3.4: Extrema, Special Theorems, First and Second Derivative Tests

1. The first derivative of the function f is given by $f'(x)=\frac{\sin^2 x}{x}-\frac{1}{2}$. What are its critical numbers on the open interval $(0, 5)$? Need to find $f'(x)=0$ or $f'(x)$ DNE

Use calculator for $f'(x)=0$

* either solve $(0=f'(x))$

or Graph & find zeros

$\boxed{x=0.555,\ 1.849}$

$f'(x)$ DNE

$x=0 \Rightarrow$ not in $(0,5)$

2. The derivative of f is $x^3(x+5)(x-1)$. At which values of x will the graph of f have a relative maximum? A relative minimum?

$f'(x)=0$

$x=0,\ -5,\ 1$

$(-\infty,-5)$	$(-5,0)$	$(0,1)$	$(1,\infty)$
$f'(x)<0$	$f'(x)>0$	$f'(x)<0$	$f'(x)>0$
min	max	min	

Relative maximum at $x=0$

Relative minimums at $x=-5$ and $x=1$

3. If $f''(x)=(x-5)(x-1)(x+2)^3$, then the graph of f has inflection points when $x=$?

$f''(x)=0$

$x=5,1,-2$

$(-\infty,-2)$	$(-2,1)$	$(1,5)$	$(5,\infty)$
$f''(x)<0$	$f''(x)>0$	$f''(x)<0$	$f''(x)>0$

inflection points at $x=-2,1,5$

4. Let $f(x)=\frac{2x^2}{x^2+4}$ on $[-5,5]$.

$f'(x)=\frac{4x(x^2+4)-2x^2(2x)}{(x^2+4)^2}=\frac{16x}{(x^2+4)^2}$

$f'(x)=0$

$x=0$

a. On what interval(s) is f increasing? Justify your conclusion.

$(-5,0)$	$(0,5)$
$f'(x)<0$	$f'(x)>0$

$f(x)$ is increasing on $(0,5)$ because $f'(x)>0$ for all x in the interval

b. On what interval(s) is f concave up? Justify your conclusion.

$f''(x)=16\left(\frac{1(x^2+4)^2-x(2(x^2+4)(2x))}{(x^2+4)^4}\right)$

$f''(x)=16(x^2+4)\left(\frac{x^2+4-4x^2}{(x^2+4)^4}\right)$

$f''(x)=\frac{16(4-3x^2)}{(x^2+4)^3}$

$f''(x)=0$

$4-3x^2=0$

$4=3x^2$

$\frac{4}{3}=x^2$

$x=\pm\frac{2}{\sqrt{3}}$

$(-\infty,\frac{-2}{\sqrt{3}})$	$(\frac{-2}{\sqrt{3}},\frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}},\infty)$
$f''(-2)<0$	$f''(0)>0$	$f''(2)<0$

Concave up on $(\frac{-2}{\sqrt{3}},\frac{2}{\sqrt{3}})$ because $f''(x)>0$ for all x in I

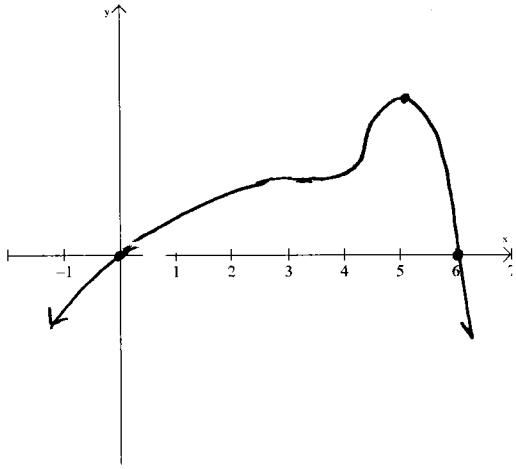
Concave down on $(-\infty,\frac{-2}{\sqrt{3}})$ & $(\frac{2}{\sqrt{3}},\infty)$ because $f''(x)<0$ for all x in I

c. At what value(s) of x does f have an inflection point? Justify your conclusion.

Points of inflection $(\pm\frac{2}{\sqrt{3}},\frac{1}{2})$ because $f''(x)$ changes from neg. to pos. at $x=\frac{-2}{\sqrt{3}}$ and from pos. to neg. at $x=\frac{2}{\sqrt{3}}$

5. Sketch a possible graph that has the following characteristics:

$f(0) = f(6) = 0$ → points $(0,0)$, $(6,0)$
$f'(3) = f'(5) = 0$ → critical numbers at $x = 3, 5$
$f'(x) > 0$ if $x < 3$ → increasing $(-\infty, 3)$
$f'(x) > 0$ if $3 < x < 5$ → increasing $(3,5)$
$f'(x) < 0$ if $x > 5$ → decreasing $(5,\infty)$
$f''(x) < 0$ if $x < 3$ and $x > 4$ → concave down $(-\infty,3)$, $(4,\infty)$
$f''(x) > 0$, $3 < x < 4$ → concave up $(3,4)$



6. Let f be the function given by $f(x) = -x^{2/3}$. Which of the following statements about f are true?

$f'(x) = \frac{-2}{3x^{1/3}}$
↖ $f'(x)$ DNE at $x=0$

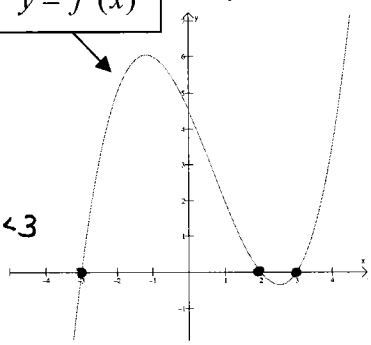
I. f is continuous at $x = 0$
II. f is differentiable at $x = 0$.
III. f has an absolute maximum at $x = 0$.

$y = x^{2/3}$ is a graph for which you should be familiar

$y = -x^{2/3}$ is the reflection over the x-axis

7. The graph of the derivative of the function, $f(x)$, is shown at the right.

$y = f'(x)$



a. Locate the x-value(s) where there is a relative maximum. Justify.

$x = 2$

since $f'(2) = 0$ & $f'(x) > 0$ when $-3 < x < 2$, $f'(x) < 0$ when $2 < x < 3$

b. Locate the x-value(s) where there is a relative minimum. Justify.

$x = -3$ → $f'(-3) = 0$, $f'(x) < 0$ for $(-\infty, -3)$, $f'(x) > 0$ for $(-3, 2)$

$x = 3$ → $f'(3) = 0$, $f'(x) < 0$ for $(2, 3)$, $f'(x) > 0$ for $(3, \infty)$

critical numbers occur when $f'(x) = 0$ or $f'(x)$ DNE

$x = -3, 2, 3$

max occurs when $f'(x)$ changes from + to −

min occurs when $f'(x)$ changes from − to +