This Teacher Resource Guide, revised in July 2018, provides supporting materials to help educators successfully implement the Indiana Academic Standards for Calculus. This resource guide is provided to help ensure all students meet the rigorous learning expectations set by the academic standards. Use of this guide and the resources on the web page is optional – teachers should decide which resources will work best for their students. However, all guidance contained in this document and on the website has been chosen to best support effective teaching practices and promote the Mathematics Process Standards.

With an increased emphasis on content area literacy, academic vocabulary has been noted. Best practices should be utilized when teaching students academic vocabulary. Please see the Literacy Framework and the Science and Technical Subjects Content Area Literacy Standards for examples of best practices.

Examples have been removed from the document as they tend to limit interpretation and classroom application. Rather, success criteria, in the form of "I can" statements, have been included. According to Hattie (2017), success criteria is specific, concrete and measurable, describing what success looks like when a learning goal is reached. Additionally, success criteria contributes to teacher clarity, which has a 0.75 effect size! An effect size of 0.40 reportedly indicates one year of growth. Utilizing success criteria in the classroom allows students to monitor their own learning and increases motivation (Hattie, p. 57). **It is important to note that the success criteria provided here are not intended to be limiting. Teachers may have additional success criteria for their students.**

Guidance around vertical articulation has been provided in the last two columns. Knowing what was expected of students at previous grade levels will help teachers connect new learning to prior knowledge. Additionally, understanding what a student will be expected to learn in the future provides the teacher a context for the current learning. This information is not exhaustive; rather it is provided to give teachers a quick understanding of how the work builds from previous grade levels into subsequent courses. The Indiana Department of Education (IDOE) math team recommends teachers further study this vertical articulation to situate their course objectives in the broader math context.

If you have any questions, please do not hesitate to reach out to the IDOE math team. Contact information for the Elementary and Secondary Math Specialists can be found on the website: https://www.doe.in.gov/standards/mathematics. If you have suggested resources for the website, please share those as well.

Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics: What works best to optimize student learning, grades K-12*. Thousand Oaks, CA: Corwin Mathematics.

| Limits and Continuity | | | | |
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| **Calculus Mathematics Standards** | | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| MA.C.LC.1: | Understand the concept of limit and estimate limits from graphs and tables of values. | I can explain the concept of the limit of a function.<br><br>I can estimate the limit of a function from a graph.<br><br>I can estimate the limit of a function from a table.<br><br>I can describe how a limit can fail to exist. | Limit | Decide if a function is continuous at a point. (MA.PC.F.7)<br><br>Describe the concept of the limit of a sequence and a limit of a function. (MA.PC.F.10) |
| MA.C.LC.2: | Find limits by substitution. | I can find limits of functions using substitution. | Limit | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.LC.3: | Find limits of sums, differences, products, and quotients. | I can find limits of sums, differences, products, and quotients of functions. | Limit | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.LC.4: | Find limits of rational functions that are undefined at a point. | I can find limits of functions that are undefined at a point. | Limit<br><br>Rational function<br><br>Undefined | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.LC.5: | Find limits at infinity. | I can find limits at infinity. | Limit | **A thorough understanding of** |

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| MA.C.LC.6: | Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. | I can decide when a limit is infinite.<br><br>I can use limits involving infinity in order to describe asymptotic behavior.<br><br>I can evaluate infinite limits and use them to define vertical asymptotes.<br><br>I can use limits involving infinity to describe end behavior. | Limit<br><br>Asymptotic behavior<br><br>Vertical asymptote<br><br>End behavior | Graph relations and functions; identify and describe features, such as end behavior. (MA.AII.PR.2)<br><br>Graph rational functions; identify and describe features, such as asymptotic and end behavior. (MA.PC.QPR.2) |
| MA.C.LC.7 | Find one-sided limits. | I can find one-sided limits.<br><br>I can describe the difference between a one-sided and a two-sided limit. | Limit<br><br>One-sided limit | Decide if a function is continuous at a point. (MA.PC.F.7) |
| MA.C.LC.8: | Understand continuity in terms of limits. | I can explain continuity in terms of limits.<br><br>I can investigate continuity at a point. | Continuity<br><br>Limit | Decide if a function is continuous at a point; use the concept of limits to describe discontinuity. (MA.PC.F.7) |
| MA.C.LC.9: | Decide if a function is continuous at a point. | I can determine if a function is continuous at a specific point. | Continuity<br><br>Discontinuity | Decide if a function is continuous at a point. (MA.PC.F.7) |

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| | | I can identify the continuity of a function. | | |
| MA.C.LC.10: | Find the types of discontinuities of a function. | I can determine the type of discontinuity found at a point of a function.<br><br>I can identify removable and non-removable discontinuities. | Discontinuity<br><br>Continuity<br><br>Removable discontinuity<br><br>Jump discontinuity<br><br>Infinite discontinuity | Find the types of discontinuities of a function and relate them to finding limits of a function. (MA.PC.F.7) |
| MA.C.LC.11: | Understand and use the Intermediate Value Theorem on a function over a closed interval. | I can explain the Intermediate Value Theorem.<br><br>I can determine if a function is continuous in order to apply the Intermediate Value Theorem.<br><br>I can use the Intermediate Value Theorem over a closed interval. | Intermediate Value Theorem<br><br>Closed interval<br><br>Continuous function | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.LC.12: | Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval. | I can understand the Extreme Value Theorem.<br><br>I can apply the Extreme Value Theorem over a closed interval. | Extreme Value Theorem<br><br>Relative minimum<br><br>Relative maximum<br><br>Absolute minimum | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| | | I can determine whether the Mean Value Theorem is applicable. | Absolute maximum<br><br>Minimum value<br><br>Maximum value | |

| Differentiation | | | | |
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| **Calculus Mathematics Standards** | | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| MA.C.D.1: | Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change. | I can interpret the derivative as a rate of change.<br><br>I can describe the derivative at a specific point of a function given the graph of that function.<br><br>I can describe the derivative at a specific point of a function given its equation.<br><br>I can describe the derivative at a specific point of a function given a table of values. | Tangent line<br><br>Derivative<br><br>Rate of change | Define slope as vertical change for each unit of horizontal change and recognize that a constant rate of change or constant slope describes a linear function. (MA.7.AF.4) |

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| MA.C.D.2: | State, understand, and apply the definition of derivative. | I can state the formal definition of a derivative.<br><br>I can analyze the definition of a derivative and explain the how the formula was derived.<br><br>I can find the derivative of a function at a point using the definition of a derivative. | Definition of a derivative<br><br>Limit definition of a derivative | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.3: | Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions. | I can differentiate basic functions, including polynomial and trigonometric functions not only using the limit definition of the derivative, but also using derivative rules.<br><br>I can compute derivatives of logarithmic and exponential functions. | Derivative<br><br>Algebraic function<br><br>Trigonometric function<br><br>Logarithmic function<br><br>Exponential function | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.4: | Find the derivatives of sums, products, and quotients. | I can differentiate functions involving sums, products, and quotients. | Derivative | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.5: | Find the derivatives of composite functions, using the chain rule. | I can perform the Chain Rule to differentiate a composite function. | Derivative<br><br>Chain rule | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| MA.C.D.6: | Find the derivatives of implicitly-defined functions. | I can find the derivative of a function defined implicitly. | Derivative<br><br>Implicit differentiation. | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.7: | Find the derivatives of inverse functions. | I can find the derivative of the inverse of a function. | Derivative<br><br>Inverse function | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.8: | Find second derivatives and derivatives of higher order. | I can use differentiation techniques to find the second derivative of a function.<br><br>I can find higher order derivatives. | Second derivative<br><br>Differentiation | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.9: | Find derivatives using logarithmic differentiation. | I can state the properties of natural logs in order to use logarithmic differentiation.<br><br>I can identify functions that would require logarithmic differentiation.<br><br>I can find the derivatives of more complicated functions using logarithmic differentiation. | Logarithmic differentiation<br><br>Properties of natural logarithms<br><br>Derivative | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.D.10: | Understand and apply the relationship between differentiability and continuity. | I can define differentiability at a point. | Differentiable<br><br>Continuity | **A thorough understanding of** |

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| | | I can explain that differentiability implies continuity with examples and non-examples. | | **Algebra I and Algebra II standards is necessary** |
| MA.C.D.11: | Understand and apply the Mean Value Theorem. | I can determine if the Mean Value Theorem can be applied to a function.<br><br>I can apply the Mean Value Theorem to a function to predict a particular slope over an interval.<br><br>I can describe the Mean Value Theorem in terms of slopes of secants and tangents being equivalent over an interval. | Mean Value Theorem<br><br>Slope of secant<br><br>Slope of tangent | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

| **Application of the Derivative** | | | |
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| **Calculus Mathematics Standards** | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |

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| MA.C.AD.1: | Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents. | I can find the slope of a curve at a point.<br><br>I can find the slope of a point that has a vertical tangent.<br><br>I can find the slope of a point that has a horizontal tangent. | Tangent line | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AD.2: | Find a tangent line to a curve at a point and a local linear approximation. | I can calculate the equation of the tangent line to a curve at a specific point.<br><br>I can use local linear approximation to estimate values on the curve.<br>I can find the tangent line approximations of a function at a given x value. | Local linear approximation<br><br>Tangent line | Represent linear functions as equations from a given point on a line and the slope of the line. (MA.AI.L.4) |
| MA.C.AD.3: | Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f'. | I can identify intervals of increase and decrease of a function.<br><br>I can describe the relationship between the sign of the derivative of a function at a point and whether or not the function itself is increasing or decreasing.<br><br>I can predict when a function is increasing or decreasing from the sign of f'. | Increasing function<br><br>Decreasing function | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, including where the function is increasing or decreasing. (MA.PC.F.1) |

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| MA.C.AD.4: | Solve real-world and other mathematical problems finding local and absolute maximum and minimum points with and without technology. | I can use derivatives to find local and absolute extrema without technology.<br><br>I can use technology to find local and absolute extrema.<br><br>I can solve real-world problems requiring differentiation. | Derivative<br><br>Relative (local) minimum<br><br>Relative (local) maximum<br><br>Absolute minimum<br><br>Absolute maximum<br><br>Minimum value<br><br>Maximum value | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, including relative maximums and minimums. (MA.PC.F.1) |
| MA.C.AD.5: | Analyze real-world problems modeled by curves, including the notions of monotonicity and concavity with and without technology. | I can analyze a curve in terms of monotonicity using derivatives.<br><br>I can describe the concavity of a curve using derivatives.<br><br>I can analyze a curve in terms of monotonicity and concavity using technology.<br><br>I can interpret the meaning of the presence or absence of monotonicity and concavity of a real-world function. | Monotonicity<br><br>Concavity<br><br>Derivative | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| MA.C.AD.6: | Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f". Understand points of inflection as places where concavity changes. | I can identify points of inflection on a graph.<br><br>I can explain concavity and its relationship to the sign of the second derivative.<br><br>I can interpret a second derivative of zero as being the point where the concavity of a function changes.<br><br>I can use the second derivative to determine intervals of concavity. | Point of Inflection<br><br>Second derivative | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AD.7: | Use first and second derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology. Compare the corresponding characteristics of the graphs of f, f', and f". | I can use the first derivative test to find and classify critical points.<br><br>I can use the second derivative test to find and verify points of inflection.<br><br>I can find critical points and inflection points using derivatives in order to sketch a graph that models a real-world problem.<br><br>I can sketch the graph of a derivative function given the function.<br><br>I can sketch a function given its derivative graph. | First derivative test<br><br>Second derivative test<br><br>Critical point<br><br>Point of inflection | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| | | I can sketch a function given its second derivative graph.<br><br>I can sketch a second derivative graph given a function.<br><br>I can determine and describe the original function by analyzing its first and second derivatives. | | |
| MA.C.AD.8: | Use implicit differentiation to find the derivative of an inverse function. | I can use implicit differentiation to find the derivative of the inverse of a function. | Implicit differentiation | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AD.9: | Solve optimization real-world problems with and without technology. | I can solve real-world optimization problems with technology.<br><br>I can identify the quantity to be optimized and the constraint.<br><br>I can write the objective (optimization) and constraint equations.<br><br>I can solve real-world optimization problems using differentiation.<br><br>I can use extrema to solve real-world optimization problems. | Optimization<br><br>Constraint equation<br><br>Objective function (optimization function) | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| MA.C.AD.10: | Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including distance, velocity, and acceleration. | I can find average rate of change.<br><br>I can find instantaneous rate of change.<br><br>I can use derivatives to describe rate of change at a point, and to compare it to average rate of change between two points.<br><br>I can define instantaneous rate of change as the limit of the average rate of change.<br><br>I can find a derivative using the limit definition of the derivative.<br><br>I can describe the connection among position, velocity, and acceleration. | Average rate of change<br><br>Instantaneous rate of change | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AD.11: | Find the velocity and acceleration of a particle moving in a straight line. | I can find the velocity of a particle moving in a straight line using derivatives.<br><br>I can interpret the derivative as velocity of a position function.<br><br>I can interpret the second derivative as acceleration of a position function. | Velocity<br><br>Acceleration | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| | | I can find the acceleration of a particle moving in a straight line using derivatives. | | |
| MA.C.AD.12: | Model rates of change, including related rates problems. | I can draw a picture of a given situation modeling rates of change.<br><br>I can identify how various rates are related in a problem and write an equation.<br><br>I can use differentiation to solve related rates problems.<br><br>I can use related rates to solve real-world problems. | Related rates<br><br>Differentiation | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

| **Integrals** | | | | |
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| **Calculus Mathematics Standards** | | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| MA.C.I.1: | Use rectangle approximations to find approximate values of integrals. | I can use rectangles to approximate the area under a curve. | Integral | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.I.2: | Calculate the values of Riemann Sums over equal subdivisions using | I can calculate Riemann Sums using left-hand and right-hand sums. | Riemann Sum<br><br>Left-hand sum | **A thorough understanding of** |

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| | left, right, and midpoint evaluation points. | I can calculate Riemann Sums using midpoints. | Right-hand sum<br><br>Midpoint | **Algebra I and Algebra II standards is necessary** |
| MA.C.I.3: | Interpret a definite integral as a limit of Riemann Sums. | I can identify a definite integral as a limit of Riemann Sums.<br><br>I can write a Riemann sum equivalent to a definite integral. | Definite integral<br><br>Riemann Sums | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.I.4: | Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $\int_a^b f'(x)dx = f(b) - f(a)$. | I can explain the Fundamental Theorem of Calculus.<br><br>I can relate the total change in a quantity as being equivalent to the definite integral of the rate of change of that quantity. | Fundamental Theorem of Calculus<br><br>Definite integral<br><br>Rate of change | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.I.5: | Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined. | I can use the Fundamental Theorem of Calculus to evaluate definite integrals.<br><br>I can use the Fundamental Theorem of Calculus to evaluate indefinite integrals.<br><br>I can represent antiderivatives using the Fundamental Theorem of Calculus. | Fundamental Theorem of Calculus<br><br>Definite integral<br><br>Indefinite integral<br><br>Antiderivative | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| | | I can represent the Fundamental Theorem of Calculus both analytically and graphically. | | |
| MA.C.I.6: | Understand and use these properties of definite integrals.<br>$\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$<br>$\int_a^b k f(x)dx = k\int_a^b f(x)dx$<br>$\int_a^a f(x)dx = 0$<br>$\int_a^b f(x)dx = -\int_b^a f(x)dx$<br>$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$<br>$If\ f(x) \le g(x)\ on\ [a,b],\ then\ \int_a^b f(x)dx \le \int_a^b g(x)dx$ | I can apply the properties of definite integrals to mathematical problems. | | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.I.7: | Understand and use integration by substitution (or change of variable) to find values of integrals. | I can discuss integration by substitution as a method to find values of integrals where the integration is apparently difficult.<br><br>I can use a change of variables (u-substitution) to evaluate integrals. | Integration by substitution | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.I.8: | Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values. | I can explain Riemann Sums as a method to approximate area under the curve.<br><br>I can discuss the Trapezoidal Rule as a method to approximate area under the curve.<br><br>I can use Riemann Sums and the Trapezoidal Rule to approximate the | Trapezoidal Rule | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| | | area under a curve of functions represented algebraically, geometrically, and by tables of values.<br><br>I can use technology to estimate the area under a curve of functions represented algebraically, geometrically, and by tables of values. | | |

| **Application of Integrals** | | | | |
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| **Calculus Mathematics Standards** | | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| MA.C.AI.1: | Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line. | I can find the specific antiderivative when given the initial condition.<br><br>I can evaluate definite and indefinite integrals and incorporate initial conditions to evaluate the constant of integration.<br><br>I can find a velocity function when given the acceleration function.<br><br>I can find a position function when given a velocity function. | Antiderivative<br><br>Definite integral<br><br>Indefinite integral<br><br>Constant of integration<br><br>Initial condition<br><br>Velocity<br><br>Acceleration | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| MA.C.AI.2: | Solve separable differential equations and use them in modeling real-world problems with and without technology. | I can solve separable differential equations.<br><br>I can use separable differential equations to model real-world problems with technology.<br><br>I can use separable differential equations to model real-world problems without technology. | Separable differential equation | |
| MA.C.AI.3: | Solve differential equations of the form y' = ky as applied to growth and decay problems. | I can solve problems where the rate of growth is proportional to the amount present. | Differential equation<br><br>Proportional<br><br>Growth/decay | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AI.4: | Use definite integrals to find the area between a curve and the x-axis, or between two curves. | I can use definite integrals to find the area between a curve and the x-axis.<br><br>I can use definite integrals to find the area bound by two curves.<br><br>I can find the limits of integration when finding the area bound by two curves. | Definite integral<br><br>Limits of integration | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AI.5: | Use definite integrals to find the average value of a function over a closed interval. | I can use definite integrals to find the average value of a function over a closed interval | Average value<br><br>Closed interval | **A thorough understanding of Algebra I and Algebra II standards is necessary** |

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| | | | Definite integral | |
| MA.C.AI.6: | Use definite integrals to find the volume of a solid with known cross-sectional area. | I can use definite integrals to find the volume of a solid with known cross-sectional area.<br><br>I can find the volume of a solid that is obtained by revolving a plane region about a horizontal or vertical line that does not pass through the plane.<br><br>I can determine if the plane is made up of disks, washers, or cylindrical shells. | Cross-sectional area<br><br>Definite integral | **A thorough understanding of Algebra I and Algebra II standards is necessary** |
| MA.C.AI.7: | Apply integration to model and solve (with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up and approximating Riemann Sum and representing its limit as a definite integral. | I can use integration techniques to solve real-world problems with technology.<br><br>I can use integration techniques to solve real-world problems without technology. | Integration techniques<br><br>Riemann sums<br><br>Limit<br><br>Definite integral<br><br>Rate of change | **A thorough understanding of Algebra I and Algebra II standards is necessary** |