This Teacher Resource Guide, revised in July 2018, provides supporting materials to help educators successfully implement the Indiana Academic Standards for Quantitative Reasoning.  This resource guide is provided to help ensure all students meet the rigorous learning expectations set by the academic standards.  Use of this guide and the resources on the web page is optional – teachers should decide which resources will work best for their students. However, all guidance contained in this document and on the website has been chosen to best support effective teaching practices and promote the Mathematics Process Standards.

With an increased emphasis on content area literacy, academic vocabulary has been noted. Best practices should be utilized when teaching students academic vocabulary.  Please see the Literacy Framework and the Science and Technical Subjects Content Area Literacy Standards for examples of best practices.

Examples have been removed from the document as they tend to limit interpretation and classroom application.  Rather, success criteria, in the form of "I can" statements, have been included.  According to Hattie (2017), success criteria is specific, concrete and measurable, describing what success looks like when a learning goal is reached. Additionally, success criteria contributes to teacher clarity, which has a 0.75 effect size! An effect size of 0.40 reportedly indicates one year of growth. Utilizing success criteria in the classroom allows students to monitor their own learning and increases motivation (Hattie, p. 57). **It is important to note that the success criteria provided here are not intended to be limiting.  Teachers may have additional success criteria for their students.**
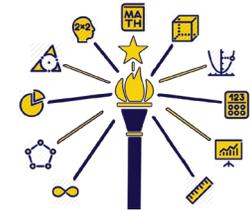
Guidance around vertical articulation has been provided in the last two columns.  Knowing what was expected of students at previous grade levels will help teachers connect new learning to prior knowledge. Additionally, understanding what a student will be expected to learn in the future provides the teacher a context for the current learning. This information is not exhaustive; rather it is provided to give teachers a quick understanding of how the work builds from previous grade levels into subsequent courses.  The Indiana Department of Education (IDOE) math team recommends teachers further study this vertical articulation to situate their course objectives in the broader math context.

If you have any questions, please do not hesitate to reach out to the IDOE math team.  Contact information for the Elementary and Secondary Math Specialists can be found on the website: https://www.doe.in.gov/standards/mathematics. If you have suggested resources for the website, please share those as well.

Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics: What works best to optimize student learning, grades K-12.* Thousand Oaks, CA: Corwin Mathematics.

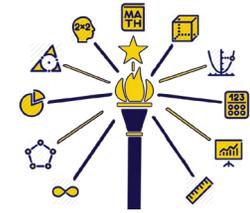| Numeracy | | | |
|---|---|---|---|
| **Quantitative Reasoning Mathematics Standards** | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| QR.N.1: Represent quantities in equivalent forms (fractions, decimals, and percentages) to investigate and describe quantitative relationships and solve real-world problems in a variety of contexts. Compare the size of numbers in different forms arising in authentic real-world contexts, such as growth expressed as a fraction versus as a percentage. Interpret the meaning of numbers in different forms, such as the meaning of a fraction or the meaning of a percentage greater than 100 and its validity in a given context. Recognize incorrect or deceptive uses of fractions, decimals, or percentages. | I can represent quantities in equivalent forms, including fractions, decimals, and percentages to describe quantitative relationships.<br><br>I can represent quantities in equivalent forms, including fractions, decimals, and percentages to solve real-world problems in a variety of contexts.<br><br>I can compare the size of numbers in different forms in a given context, such as growth expressed as a fraction versus as a percentage.<br><br>I can interpret the meaning of a fraction or the meaning of a percentage greater than 100 as describing growth in a given context.<br><br>I can assess the validity of a fraction or the meaning of a percentage greater than 100 in a given context. | Quantitative relationships | |

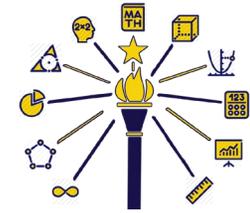| | | I can recognize incorrect or deceptive uses of fractions, decimals, or percentages. | | |
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| QR.N.2: | Solve problems involving calculations with percentages and interpret the results, such as calculating percentage rates or differentiating between a discount of 30% and two consecutive discounts of 15%. Calculate relative change and explain how it differs from absolute change. Recognize incorrect or deceptive uses of percentages. | I can solve problems involving percentages.<br><br>I can interpret the results of problems computed with percentages.<br><br>I can calculate percentage rates.<br><br>I can differentiate between a single discount and two consecutive discounts.<br><br>I can calculate relative change.<br><br>I can explain the difference between relative change and absolute change.<br><br>I can recognize incorrect or deceptive use of percentages. | Relative change<br><br>Absolute change | |
| QR.N.3: | Interpret numbers in different forms in terms of authentic contexts to solve real-world problems, such as interpreting a growth rate less than 1%. Compare and precisely communicate with numbers in different forms (including words, | I can interpret numbers in different forms in terms of a given context.<br><br>I can interpret a growth rates less than 1%. | Standard notation<br><br>Scientific notation<br><br>Relative change | Solve real-world and other mathematical problems involving numbers expressed in scientific notation. (MA.8.C.2) |

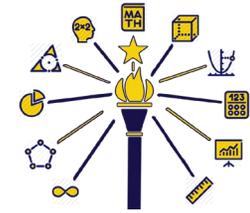| | | | | |
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| | fractions, decimals, standard notation, and scientific notation), such as comparing relative and absolute changes in quantities. | I can compare and communicate with numbers in various forms, including words, fractions, decimals and scientific notation.<br><br>I can compare relative change and absolute change in quantities. | Absolute change | |
| QR.N.4: | Compare magnitudes of numbers in context, such as the population of the US compared to the population of the world. Perform such comparisons when numbers are in different forms (including words, fractions, decimals, standard notation, and scientific notation). | I can order compare the magnitude of numbers in context.<br><br>I can compare the magnitude of numbers written in different forms, including words, fractions, decimals and scientific notation. | Magnitude of numbers<br><br>Standard notation<br><br>Scientific notation | Solve real-world and other mathematical problems involving numbers expressed in scientific notation. (MA.8.C.2) |
| QR.N.5: | Perform accurate and efficient calculations using large and small numbers in different forms, to an appropriate precision, with and without technology. Include calculations in context, such as ratios representing water use per capita for a large population. | I can accurately and efficiently make calculations using large and small numbers in different forms with and without technology.<br><br>I can accurately and efficiently make calculations using large and small numbers in context with and without technology. | Per capita | |
| QR.N.6: | Use estimation skills, and know why, how, and when to estimate results. Identify and use numeric benchmarks for estimating calculations (e.g., using 25% as an estimate for 23%). Identify and use contextual benchmarks for | I can determine why, how, and when it is appropriate to estimate.<br><br>I can identify and use numeric benchmarks to estimate. | Numeric benchmarks<br><br>Contextual benchmarks | |

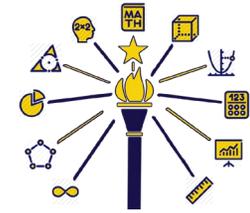| | | | |
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| | comparison to other numbers (e.g., using US population as a benchmark to evaluate reasonableness of statistical claims or giving context to numbers). Check for reasonableness using both types of benchmarks. | I can identify and use contextual benchmarks when comparing and giving context to numbers.<br><br>I can determine the reasonableness of my response using both numeric and contextual benchmarks. | | |
| QR.N.7: | Use dimensional analysis to convert between units of measurements and to solve problems involving multiple units of measurement, such as converting between currencies, calculating the cost of gasoline to drive a given car a given distance, or calculating dosages of medicine. | I can use dimensional analysis to convert between units of measurements.<br><br>I can use dimensional analysis to solve problems involving multiple units of measurement. | Dimensional analysis | Convert between measurement systems given conversion factors, and use these conversions in solving real-world problems. (MA.6.GM.1) |

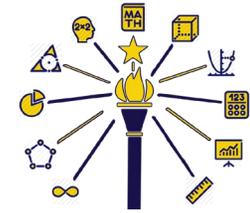| Ratio And Proportional Reasoning | | | |
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| **Quantitative Reasoning Mathematics Standards** | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| QR.RP.1: Solve real-life problems requiring interpretation and comparison of complex numeric summaries which extend beyond simple measures of center, such as problems requiring interpreting and/or comparing weighted averages, indices, coding, and ranking. Evaluate claims based on complex numeric summaries. | I can interpret and compare weighted averages.<br><br>I can interpret and compare indices.<br><br>I can interpret and compare coding and ranking number summaries. | Weighted averages<br><br>Indices<br><br>Coding<br><br>Ranking | |

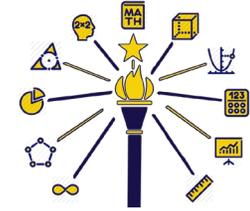| | | I can evaluate claims based on weighted averages, indices, coding, or ranking. | Complex numeric summaries | |
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| QR.RP.2: | Understand and communicate percentages as rates per 100, and identify uses and misuses of percentages related to a proper understanding of the base in real-world and mathematical problems. | I can understand and communicate percentages as rates per 100.<br><br>I can identify uses and misuses of percentages in real-world problems. | | |
| QR.RP.3: | Solve real-life problems requiring interpretation and comparison of various representations of ratios, (i.e. fractions, decimals, rate, and percentages), such as problems that involve non-standard ratios (e.g., media and risk reporting) or part-to-part versus part-to-whole ratios taken from meaningful context. | I can interpret and compare various representations of ratios, such as fractions, decimals, rates, and percentages.<br><br>I can solve real-world problems involving non-standard ratios such as media and risk reporting.<br><br>I can solve real-world problems involving non-standard ratios such as part-to-part or part-to-whole ratios. | Part-to-part ratios<br><br>Part-to-whole ratios<br><br>Non-standard ratios | |
| QR.RP.4: | Analyze growth and decay using absolute and relative change and make comparisons using absolute and relative difference. | I can analyze growth and decay using absolute change.<br><br>I can analyze growth and decay using relative change. | Absolute growth<br><br>Absolute decay<br><br>Relative growth<br><br>Relative decay | Understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals; compare linear functions and exponential functions that |

|  |  | I can compare absolute and relative differences in growth and decay problems. | Absolute difference

Relative difference | model real-world situations. (MA.AI.QE.1)

Identify the percent rate of change in exponential functions written as equations. (MA.AII.EL.3) |
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| QR.RP.5: | Distinguish between proportional and non-proportional situations, and, when appropriate, apply proportional reasoning, such as when solving for an unknown quantity in proportional situations; solving real-life problems requiring conversion of units using dimensional analysis; or applying scale factors to perform indirect measurements (e.g., maps, blueprints, concentrations, dosages, and densities). Recognize when proportional techniques do not apply. | I can determine if a situation is proportional or non-proportional.

I can apply proportional reasoning when appropriate and when proportional techniques do not apply.

I can solve real-world problems requiring conversion of units using dimensional analysis.

I can apply scale factors to perform indirect measurements. | Proportional situations

Non-proportional situations

Proportional reasoning

Dimensional analysis

Scale factor

Indirect measurement | Convert between measurement systems given conversion factors, and use these conversions in solving real-world problems. (MA.6.GM.1)

Decide whether two quantities are in a proportional relationship. (MA.7.AF.6)

Identify real-world and other mathematical situations that involve proportional relationships. (MA.7.AF.9)

Create a scale drawing by using proportional reasoning. (MA.7.GM.3) |
| QR.RP.6: | Determine the constant of proportionality in proportional situations (both real-life and mathematical), leading to a symbolic model for the situation (i.e. an | I can determine the constant of proportionality when applicable. | Constant of proportionality

Proportional situation | Identify the unit rate or constant of proportionality in tables, graphs, equations, and verbal descriptions of |
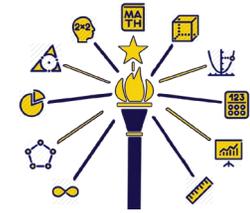
| | | | | proportional relationships. (MA.7.AF.7) |
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| | equation based upon a rate of change, **y = kx**). | I can model a proportional relationship using an equation based upon the constant of proportionality. | | Represent real-world and other mathematical problems using an algebraic proportion that leads to a linear equation. (MA.AI.L.3) |

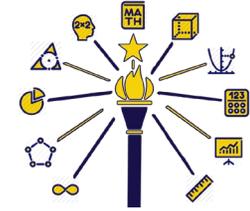| Modeling | | | | |
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| **Quantitative Reasoning Mathematics Standards** | | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| QR.M.1: | Analyze and critique mathematical models and be able to describe their limitations, including distinguishing between correlation and causation and determine whether interpolation and/or extrapolation are appropriate. | I can critique mathematical models and describe their limitations.<br><br>I can distinguish between correlation and causation.<br><br>I can justify the use of interpolation and/or extrapolation and determine their appropriateness. | Mathematical models<br><br>Correlation<br><br>Causation<br><br>Interpolation<br><br>Extrapolation | Use technology to find a function that models a relationship for a bivariate data set to make predictions; interpret the correlation coefficient. (MA.AII.DSP.2) |
| QR.M.2: | Use models, including models created with spreadsheets or other tools, to estimate solutions to contextual questions, such as functional models to estimate future population or spreadsheets to model financial applications (e.g. credit card debt, installment savings, | I can use models, including those from spreadsheets, to estimate solutions to contextual questions.<br><br>I can use functional models to estimate population growth or decay. | Functional model<br><br>Parameter | Use technology to find a function that models a relationship for a bivariate data set to make predictions. (MA.AII.DSP.2) |

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| | amortization schedules, mortgage and other loan scenarios). Identify patterns and identify how changing parameters affect the results. | I can use spreadsheets to model financial applications.<br><br>I can identify patterns in the models I am using.<br><br>I can recognize how changing parameters in a model affects the results. | | |
| QR.M.3: | Choose and create, with and without technology, linear, exponential, logistic, or periodic models and curves of best fit for bivariate data sets. Use the models to answer questions and draw conclusions or make decisions, addressing limitations and long-term ramifications of chosen models when appropriate. Recognize when a change in model is needed. | I can select an appropriate model (linear, exponential, logistic, or periodic) to best fit bivariate data.<br><br>I can create, with and without technology) an appropriate model (linear, exponential, logistic, or periodic) to best fit bivariate data.<br><br>I can use my model to draw conclusions and make decisions about bivariate data.<br><br>I can address both limitations and long-term ramifications of my chosen model.<br><br>I can recognize then a change in model is needed for bivariate data. | Linear model<br><br>Exponential model<br><br>Logistic model<br><br>Periodic model<br><br>Bivariate data | Use technology to find a linear, quadratic, or exponential function that models a relationship for a bivariate data set to make predictions; compute (using technology) and interpret the correlation coefficient. (MA.AII.DSP.2) |

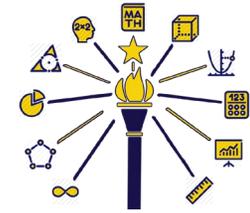| QR.M.4: | Analyze real-world problem situations and use variables to construct and solve equations involving one or more unknown or variable quantities to answer questions about the situations, such as creating spreadsheet formulas to calculate prices based on percentage mark-up or solving formulas for specified values. Demonstrate understanding of the meaning of a solution. Identify when there is insufficient information given to solve a problem. | I can analyze real-world problems, using variables to write and solve equations involving one or more unknown quantities.<br><br>I can isolate specified variables and values in formulas.<br><br>I can explain the meaning of my solution when analyzing real-world problems.<br><br>I can recognize when there is insufficient information available when solving problems. | | Solve equations and formulas for a specified variable. (MA.AI.L.11) |
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| QR.M.5: | Apply geometric concepts to model situations and solve problems such as those arising in art, architecture, and other fields. | I can apply geometric concepts to model situations.<br><br>I can solve geometric problems such as those arising in art and architecture. | | |
| QR.M.6: | The student uses a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems, such as in scheduling or routing situations that can be modeled using different methods, e.g., vertex-edge graphs using critical paths, Euler paths, or minimal spanning trees. | I can use networks to organize data in quantitative situations.<br><br>I can use networks make informed decisions and solve problems such as scheduling or routing situations.<br><br>I can read and create a vertex-edge graph using critical paths. | Vertex-edge graph<br><br>Critical path<br><br>Euler path<br><br>Minimal spanning trees | |

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| | | I can read and create a vertex-edge graph using an Euler path.<br><br>I can read and create a vertex-edge graph using the minimal spanning trees method. | | |

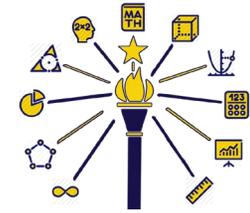| Probabilistic Reasoning to Assess Risk | | | | |
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| **Quantitative Reasoning Mathematics Standards** | | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |
| QR.P.1: | Determine the nature and number of elements in a finite sample space to model the outcomes of real-world events using counting techniques, and build the sample space by making lists, tables, or tree diagrams. | I can determine the nature of elements in a finite sample space to model the outcomes of real-world events.<br><br>I can determine the number of elements in a finite sample space to model the outcomes of real-world events using counting techniques.<br><br>I can correctly define a situation as a permutation or combination in order to determine the number of elements in a finite sample space.<br><br>I can determine the number of elements in a finite sample space to | Element<br><br>Sample space<br><br>Counting techniques<br><br>The Fundamental Counting Principle<br><br>Permutations<br><br>Combinations<br><br>Tree diagram | Represent sample spaces of compound events using organized lists, tables, and tree diagrams. (MA.8.DSP.5) |

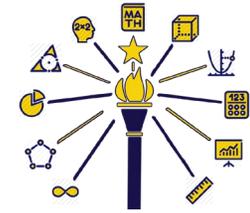| | | model the outcomes of real-world events by making lists, tables, or tree diagrams. | | |
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| QR.P.2: | Determine the number of ways an event may occur using the Fundamental Counting Principle. | I can compute the number of ways an event may occur using the Fundamental Counting Principle. | Event<br><br>Fundamental Counting Principle | Develop and apply the multiplication counting principle. (MA.8.DSP.6) |
| QR.P.3: | Evaluate the validity of claims based on empirical, theoretical, and subjective probabilities. Draw conclusions or make decisions related to risk, pay-off, expected value, and false negatives/positives in various probabilistic contexts. | I can evaluate the validity of a claim made based on an empirical probability.<br><br>I can evaluate the validity of a claim made based on a theoretical probability.<br><br>I can evaluate the validity of a claim made based on a subjective probability.<br><br>I can draw conclusions and make decisions related to risk.<br><br>I can draw conclusions and make decisions related to pay-off.<br><br>I can draw conclusions and make decisions related to expected value. | Empirical probability<br><br>Theoretical probability<br><br>Subjective probability<br><br>Risk<br><br>Pay-off<br><br>Expected value<br><br>False negative<br><br>False positive | Record multiple observations (or simulated samples) of random events and construct empirical models of the probability distributions; construct a theoretical model. (MA.AII.DSP.4) |

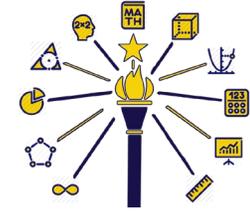| | | I can draw conclusions and make decisions related to false negatives/false positive. | | |
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| QR.P.4: | Use data displays and models, such as two-way tables, tree diagrams, Venn diagrams, and area models, to determine probabilities (including conditional probabilities) and use these probabilities to make informed decisions. | I can use two-way tables to determine probabilities and make decisions.<br><br>I can use tree diagrams to determine probabilities and make decisions.<br><br>I can use Venn diagrams to determine probabilities and make decisions.<br><br>I can use area models to determine probabilities and make decisions.<br><br>I can use data displays and models to determine conditional probabilities and make decisions. | Data display<br><br>Two-way table<br><br>Tree diagram<br><br>Venn diagram<br><br>Area model<br><br>Conditional probability | Understand dependent and independent events, and conditional probability; apply these concepts to calculate probabilities. (MA.AII.DSP.5) |

| Statistics | | | |
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| **Quantitative Reasoning Mathematics Standards** | **Success Criteria** | **Academic Vocabulary** | **Looking Back** |

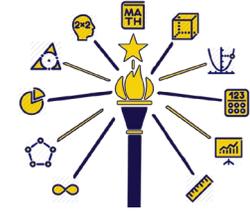| QR.S.1: | Analyze statistical information from studies, surveys, and polls (including when reported in condensed form or using summary statistics) to make informed judgments as to the validity of claims or conclusions, such as when interpreting and comparing the results of polls using margin of error. | I can analyze statistical information from studies, surveys, and polls.<br><br>I can analyze statistical information from studies, surveys, and polls when reported using summary statistics.<br><br>I can make informed judgements as to the validity of claims resulting from studies, surveys, and polls.<br><br>I can interpret the results of polls using margin of error.<br><br>I can compare the results of polls using margin of error. | Summary statistics<br><br>Margin of error | Understand that statistics can be used to gain information about a population; understand that random sampling tends to produce representative samples and support valid inferences. (MA.7.DSP.1) |
| QR.S.2: | Identify limitations, strengths, or lack of information in studies, including data collection methods (e.g. sampling, experimental, observational) and possible sources of bias, and identify errors or misuses of statistics to justify particular conclusions. | I can identify limitations, strengths, or lack of information in studies.<br><br>I can identify limitations, strengths, or lack of information in data collection methods.<br><br>I can identify possible sources of bias in studies.<br><br>I can identify errors in misuses of statistics to justify conclusions. | Sampling data collection<br><br>Experimental data collection<br><br>Observational data collection<br><br>Bias | Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (MA.AII.DSP.1) |

| QR.S.3: | Create (with and without technology) and use visual displays of real world data, such as charts, tables and graphs. | I can create without technology visual displays of real world data, such as charts, tables, and graphs.<br><br>I can create with technology visual displays of real world data, such as charts, tables, and graphs.<br><br>I can use visual displays of real world data, such as charts, tables, and graphs to answer questions. | | Organize and graph univariate data. (MA.AII.DSP.3) |
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| QR.S.4: | Interpret and analyze visual representations of data, and describe strengths, limitations, and fallacies of various graphical displays. | I can interpret visual representations of data.<br><br>I can analyze visual representations of data.<br><br>I can describe strengths, limitations, and fallacies of graphical displays of data. | Fallacy | |
| QR.S.5: | Read, interpret, and make decisions about data summarized numerically using measures of center and spread, in tables, and in graphical displays (line graphs, bar graphs, scatterplots, and histograms), e.g., explain why the mean may not represent a typical salary; explain the difference between bar graphs and histograms; critique a graphical display by recognizing that | I can read, interpret, and make decisions about data summarized using measures of center and spread in tables.<br><br>I can read, interpret, and make decisions about data summarized using measures of center and spread in line graphs. | Measures of center<br><br>Mean<br><br>Median<br><br>Mode<br><br>Measures of spread | Organize, graph (e.g., line plots and box plots), and compare univariate data of two or more different data sets using measures of center (mean and median) and spread (range, inter-quartile range, standard deviation, percentiles, and variance). (MA.AII.DSP.3) |

| | | | | |
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| | the choice of scale can distort information. | I can read, interpret, and make decisions about data summarized using measures of center and spread in bar graphs.<br><br>I can read, interpret, and make decisions about data summarized using measures of center and spread in scatterplots.<br><br>I can read, interpret, and make decisions about data summarized using measures of center and spread in histograms.<br><br>I can explain the difference between bar graphs and histograms.<br><br>I can recognize when the scale of a graph has distorted the information. | Variance<br><br>Line graph<br><br>Bar graph<br><br>Scatterplot<br><br>Histogram<br><br>Bin | |
| QR.S.6: | Summarize, represent, and interpret data sets on a single count or measurement variable using plots and statistics appropriate to the shape of the data distribution to represent it. | I can determine the appropriate statistics to use to summarize, represent, and interpret data on a single measurement variable.<br><br>I can use plots appropriately based on the shape of a single measurement variable's data distribution. | Shape of data distribution | Organize, graph (e.g., line plots and box plots), and compare univariate data of two or more different data sets. (MA.AII.DSP.3) |

| QR.S.7: | Compare center, shape, and spread of two or more data sets and interpret the differences in context. | I can compare center, shape, and spread of two or more data sets.<br><br>I can interpret the differences between two or more data sets in context. | Center<br><br>Shape<br><br>Spread<br><br>Left-skewed<br><br>Right-skewed<br><br>Unimodal<br><br>Mean<br><br>Median<br><br>Mode<br><br>Range<br><br>Variance | Organize, graph and compare univariate data of two or more different data sets using measures of center (mean and median) and spread (range, inter-quartile range, standard deviation, percentiles, and variance). (MA.AII.DSP.3) |
| --- | --- | --- | --- | --- |
| QR.S.8: | Use properties of distributions, including uniform and normal distributions, to analyze data and answer questions. | I can apply the properties of uniform distributions to analyze data and answer questions.<br><br>I can apply the properties of normal distributions to analyze data and answer questions. | Uniform distribution<br><br>Normal distribution<br><br>Standard deviation<br><br>Percentile<br><br>Z-score | Organize, graph, and compare univariate data sets using measures of center and spread, including mean, standard deviation, percentiles, and variance. (MA.AII.DSP.3) |

| | | I can find the standard deviation of normally distributed data.<br><br>I can interpret percentiles using normally distributed data.<br><br>I can determine the z-score of data points that are normally distributed. | | |
| --- | --- | --- | --- | --- |
| QR.S.9: | Recognize when data are normally distributed and use the mean and standard deviation of the data to fit it to a normal distribution. | I can identify data that is normally distributed.<br><br>I can use the mean and standard deviation of data to fit it to a normal distribution. | Normal distribution<br><br>Percentile<br><br>Mean<br><br>Standard deviation | Compare two or more univariate data sets using measures of center and spread, including mean and standard deviation. (MA.AII.DSP.3) |