This Teacher Resource Guide, revised in July 2018, provides supporting materials to help educators successfully implement the Indiana Academic Standards for Trigonometry. This resource guide is provided to help ensure all students meet the rigorous learning expectations set by the academic standards. Use of this guide and the resources on the web page is optional – teachers should decide which resources will work best for their students. However, all guidance contained in this document and on the website has been chosen to best support effective teaching practices and promote the Mathematics Process Standards.

With an increased emphasis on content area literacy, academic vocabulary has been noted. Best practices should be utilized when teaching students academic vocabulary. Please see the Literacy Framework and the Science and Technical Subjects Content Area Literacy Standards for examples of best practices.

Examples have been removed from the document as they tend to limit interpretation and classroom application. Rather, success criteria, in the form of "I can" statements, have been included. According to Hattie (2017), success criteria is specific, concrete and measurable, describing what success looks like when a learning goal is reached. Additionally, success criteria contributes to teacher clarity, which has a 0.75 effect size! An effect size of 0.40 reportedly indicates one year of growth. Utilizing success criteria in the classroom allows students to monitor their own learning and increases motivation (Hattie, p. 57). **It is important to note that the success criteria provided here are not intended to be limiting. Teachers may have additional success criteria for their students.**

Guidance around vertical articulation has been provided in the last two columns. Knowing what was expected of students at previous grade levels will help teachers connect new learning to prior knowledge. Additionally, understanding what a student will be expected to learn in the future provides the teacher a context for the current learning. This information is not exhaustive; rather it is provided to give teachers a quick understanding of how the work builds from previous grade levels into subsequent courses. The Indiana Department of Education (IDOE) math team recommends teachers further study this vertical articulation to situate their course objectives in the broader math context.

If you have any questions, please do not hesitate to reach out to the IDOE math team. Contact information for the Elementary and Secondary Math Specialists can be found on the website: https://www.doe.in.gov/standards/mathematics. If you have suggested resources for the website, please share those as well.

Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics: What works best to optimize student learning, grades K-12*. Thousand Oaks, CA: Corwin Mathematics.

Conics				
Trigonometry Mathematics Standards		**Success Criteria**	**Academic Vocabulary**	**Looking Back**
MA.TR.CO.1:	Determine how the graph of a parabola changes if a, b and c changes in the equation $y = a(x – b)^2 + c$. Find an equation for a parabola when given sufficient information.	Through exploration, I can determine how the graph of a parabola changes if a, b, and c changes in the equation $y = a(x – b)^2 + c$. I can find an equation for a parabola when given sufficient information.	Parabola	Describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k. (MA. AII.F.5)
MA.TR.CO.2:	Derive the equation of a parabola given a focus and directrix.	I can identify the vertex of a parabola as the halfway point between the focus and the directrix. I can determine the equation of a parabola given a focus and directrix.	Parabola Vertex Focus Directrix	Graph quadratic equations in two variables with and without technology. (MA.AI.QE.3)
MA.TR.CO.3:	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	I can use the Pythagorean Theorem, along with the center and radius of a circle to derive the equation of a circle. I can complete the square in order to find the center and radius of a circle given by an equation.	Center Radius Pythagorean Theorem Equation of a circle Complete the square	Prove and apply theorems about triangles, including the Pythagorean Theorem. (MA.G.T.1) Use completing the square to rewrite quadratic functions; Understand the relationship between completing the square and the quadratic formula. (MA.AII.Q.2)

MA.TR.CO.4:	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	If given the foci, I can apply the fact that the sum or difference of distances from the foci is constant to derive the equations of ellipses and hyperbolas.	Ellipse Hyperbola Foci	Derive the equation of a circle of given center and radius. (MA.TR.CO.3)
MA.TR.CO.5:	Graph conic sections. Identify and describe features like center, vertex or vertices, focus or foci, directrix, axis of symmetry, major axis, minor axis, and eccentricity.	I can graph conic sections. Given a conic section, I can identify the center, directrix, vertices, focus, and axis of symmetry. I can distinguish between the major axis and the minor axis of a conic section, where applicable. I can characterize the shape of a curve using the eccentricity of a conic section.	Conic section Center Vertex (vertices) Focus (foci) Directrix Axis of symmetry Major axis Minor axis Eccentricity	Graph exponential functions with and without technology. Identify and describe features, such as intercepts, zeros, domain and range, and asymptotic and end behavior. (MA.AII.EL.2) Graph rational functions with and without technology; identify and describe features, such as intercepts, domain and range, and asymptotic and end behavior. (MA.PC.QPR.2)

MA.TR.CO.6:	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	I can informally discuss the formulas for circumference and area of a circle. I can give an informal argument for the volume of pyramids, cylinders, and cones. I can use dissection arguments to develop geometric formulas. I can use Cavalieri's principle in both 2-dimensions and 3-dimensions. I can use Cavalieri's principle to find the volume of solids. I can use informal limit arguments to explain the formulas for the circumference of a circle, area of a circle, and volume of a pyramid, cylinder, or cone.	Circumference Cylinder Pyramid Cone Dissection argument Cavalieri's Principle Cross section Informal limit argument	Develop geometric proofs, including direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry. (MA.G.LP.4) Deduce formulas relating perimeters and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle. (MA.G.QP.5) Solve real-world and other mathematical problems involving volume of cylinders, cones, and pyramids. (MA.G.TS.5)

Unit Circle				
Trigonometry Mathematics Standards		**Success Criteria**	**Academic Vocabulary**	**Looking Back**
MA.TR.UC.1:	Understand radian measure of an angle as the length of the arc on	I can identify angles subtended by arcs.	Radian measure	Define, identify and use relationships among the

	the unit circle subtended by the angle.	I can define the angle created when the length of an arc of a circle is the same as the length of the circle's radius as a radian. I can define the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Arc Theta Central angle Unit circle	following: radius, diameter, arc, measure of an arc, and chord. (MA.G.CI.1)
MA.TR.UC.2:	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	I can recognize that the coordinates of any point on the unit circle may be defined as (cos Θ, sin Θ). I can identify that tanΘ = sinΘ/cosΘ. I can find the measure of the angle and state the trigonometric ratio for that angle when given a point on the coordinate plane. I can graph trigonometric functions on the unit circle.	Unit circle Coordinate plane Trigonometric functions Real numbers Radian measure Co-terminal angle Reference angle	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (MA.G.T.9)
MA.TR.UC.3:	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	I can define cosine and secant as being even functions and symmetric to the y-axis. I can define sine, cosecant, tangent, and cotangent as being odd and symmetric to the origin. I can compare values of trigonometric functions in quadrants I	Unit circle Odd symmetry Even symmetry Trigonometric functions	Recognize even and odd functions from their graphs. (MA.PC.F.6)

		and IV to determine whether a function is even or odd.		

Geometry				
Trigonometry Mathematics Standards		**Success Criteria**	**Academic Vocabulary**	**Looking Back**
MA.TR.G.1:	Solve real-world problems with and without technology that can be modeled using right triangles, including problems that can be modeled using trigonometric ratios. Interpret the solutions and determine whether the solutions are reasonable.	I can solve real-world problems that can be modeled using right triangles with and without technology. I can use trigonometric ratios to solve real-world problems that can be modeled with right triangles with and without technology. I can interpret the solutions to determine whether they are reasonable within the context of the problem.	Right triangle Trigonometric ratios	Use trigonometric ratios (sine, cosine and tangent) and the Pythagorean Theorem to solve real-world and mathematical problems involving right triangles. (MA.G.T.10)
MA.TR.G.2:	Explain and use the relationship between the sine and cosine of complementary angles.	I can investigate the relationship between the sine and cosine of complementary angles. I can use the idea that sine and cosine are cofunctions to solve problems.	Sine Cosine Complementary angles Cofunction	Solve real-world and other mathematical problems that involve complementary angles. (MA.7.GM.4)

MA.TR.G.3:	Use special triangles to determine the values of sine, cosine, and tangent for π/3, π/4, and π/6. Apply special right triangles to the unit circle and use them to express the values of sine, cosine, and tangent for x, π + x, and 2π – x in terms of their values for x, where x is any real number.	I can use similarity to determine the side measures of 30°-60°-90° and 45°-45°-90° triangles. I can find the values of sine, cosine, and tangent in the special right triangle using degree and radian measures. I can understand and use reference angles on the unit circle.	Special right triangles Domain Reference angle Angle of rotation Co-terminal Initial side Terminal side Sine Cosine Tangent Unit circle	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (MA.G.T.9) Use special right triangles (30° - 60° and 45° - 45°) to solve real-world and mathematical problems. (MA.G.T.11)
MA.TR.G.4:	Prove the Laws of Sines and Cosines and use them to solve problems.	I can prove the Laws of Sines and Cosines by drawing and using altitudes in both acute and obtuse triangles. I can use the Laws of Sines and Cosines to find missing angles or side lengths of triangles.	Law of sines Law of cosine Altitude Triangle Sum Theorem Complementary angles	Develop geometric proofs, including direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry. (MA.G.LP.4) Explain and justify the process used to construct altitudes, with a variety of tools. (MA.G.PL.5)

				State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. (MA.G.T.7)
MA.TR.G.5:	Understand and apply the Laws of Sines and Cosines to solve real-world and other mathematical problems involving right and non-right triangles.	I can explain the Laws of Sines and Cosines. I can determine when the use of the Laws of Sines and Cosines is necessary. I can use the Laws of Sines and Cosines to solve real-world problems.	Law of sines Law of cosines Right triangle	Prove and apply theorems about triangles, including the following: measures of interior angles of a triangle sum to 180° and the Pythagorean Theorem. (MA.G.T.1)
MA.TR.G.6:	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line. Use the formula to find areas of triangles.	I can derive the area formula for a triangle using the sine ratio. I can determine the area of a triangle given a variety of information.	Auxiliary line	Apply the area formula for a triangle. (MA.5.M.3)

Periodic Functions				
Trigonometry Mathematics Standards		**Success Criteria**	**Academic Vocabulary**	**Looking Back**
MA.TR.PF.1:	Find a sinusoidal function to model a data set and explain the parameters of the model.	I can find a sinusoidal function to model a data set.	Sinusoidal function parameter	Represent real-world problems that can be modeled with a linear function. (MA.AI.L.5)

		I can explain the parameters of a sinusoidal function that models a data set.		Represent real-world and other mathematical problems that can be modeled with exponential functions. (MA.AI.QE.2) Describe the effect on the graph of f(x) by replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k**.** (MA.PC.F.6) Model and solve real-world problems involving applications of sequences and series and interpret the solutions. (MA.PC.F.8)
MA.TR.PF.2:	Graph trigonometric functions with and without technology. Use the graphs to model and analyze periodic phenomena, stating amplitude, period, frequency, phase shift, and midline (vertical shift).	I can graph trigonometric functions with technology. I can graph trigonometric functions without technology. I can identify amplitude, frequency, period, and midline given a graph of a trigonometric function.	Trigonometric functions Periodic phenomena Amplitude Period Frequency Phase shift Midline	Graph exponential functions with and without technology. Identify and describe features, such as intercepts, zeros, domain and range, and asymptotic and end behavior. (MA.AII.EL.2) Graph rational functions with and without technology; identify and describe features, such as intercepts, domain and range, and asymptotic and

				end behavior. (MA.PC.QPR.2) Graph and solve real-world and other mathematical problems that can be modeled using exponential and logarithmic equations and inequalities. (MA.PC.EL.3)
MA.TR.PF.3:	Construct the inverse trigonometric functions of sine, cosine, and tangent by restricting the domain.	I can restrict a domain in order to construct the inverse functions of since, cosine, and tangent.	Inverse trigonometric functions Sine Cosine Tangent Domain	Produce an invertible function from a non-invertible function by restricting the domain. (MA.PC.F.5)
MA.TR.PF.4:	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	I can use inverse functions to solve trigonometric equations that arise in modeling contexts. I can interpret the solution using technology and determine if there are additional solutions. I can interpret the solution in terms of the context of the model.	Inverse function Trigonometric equations Displacement Wave speed Frequency Harmonic motion	Know that the inverse of an exponential function is a logarithmic function; describe their inverse relationship. (MA.AII.EL.5)

			Midline Equilibrium Periodic Sinusoidal	
MA.TR.PF.5:	Prove the addition and subtraction formulas for sine, cosine, and tangent. Use the formulas to solve problems.	I can prove the addition and subtraction formulas for sine, cosine, and tangent. I can use the addition and subtraction formulas for sine, cosine, and tangent to solve problems.	Sine Cosine Tangent	**Although this is an introductory standard for this concept, strong algebraic skills, computation skills, and number sense are needed.**
MA.TR.PF.6:	Prove the double- and half-angle formulas for sine, cosine, and tangent. Use the formulas to solve problems.	I can prove the double-angle formula for sine, cosine, and tangent using the addition formulas. I can prove the half-angle formula for sine, cosine, and tangent. I can use the double- and half-angle formulas for sine, cosine, and tangent to solve problems.	Double-angle formula Half-angle formula Sine Cosine Tangent	**Although this is an introductory standard for this concept, strong algebraic skills, computation skills, and number sense are needed.**
MA.TR.PF.7:	Define and use the trigonometric ratios (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles and the coordinates on the unit circle.	I can define all trigonometric ratios and their inverses in terms of angles of right triangles and the coordinates on the unit circle.	Trigonometric ratios Sine Cosine Tangent	Use trigonometric ratios (sine, cosine and tangent) and the Pythagorean Theorem to solve real-world and mathematical problems involving right triangles. (MA.G.T.10)

		I can use the unit circle and the six trigonometric ratios to solve problems.	Cotangent Secant Cosecant Right triangles Unit circle	

Identities				
Trigonometry Mathematics Standards		**Success Criteria**	**Academic Vocabulary**	**Looking Back**
MA.TR.ID.1:	Prove the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ and use it to find trigonometric ratios, given sin(x), cos(x), or tan(x), and the quadrant of the angle.	I can use the Pythagorean Theorem to prove the Pythagorean identities. I can use Pythagorean identities to find trigonometric ratios given sine, cosine, or tangent and the quadrant of the angle.	Pythagorean identify Trigonometric ratios Sine Cosine Tangent Quadrant	Use inductive reasoning to explain the Pythagorean Theorem. (MA.8.GM.7) Develop geometric proofs, including direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry. (MA.G.LP.4)
MA.TR.ID.2:	Verify basic trigonometric identities and simplify expressions using these and other trigonometric identities.	I can validate basic trigonometric identities. I can simplify expressions using basic trigonometric identities.	Trigonometric identities	**Although this is an introductory standard for this concept, strong algebraic skills, computation skills, and**

		I can use a variety of trigonometric identities to simplify expressions.		**number sense are needed.**

Polar Coordinates				
Trigonometry Mathematics Standards		**Success Criteria**	**Academic Vocabulary**	**Looking Back**
MA.TR.PC.1:	Define polar coordinates and relate polar coordinates to Cartesian coordinates.	I can define polar coordinates simplistically as a point marked by how far away from the origin and what angle it is with the x axis. I can relate polar coordinates to Cartesian coordinates.	Polar coordinates Cartesian coordinates	Understand and use complex numbers, including real and imaginary numbers, on the complex plane in rectangular and polar form. (MA.PC.PCN.2)
MA.TR.PC.2:	Translate equations from rectangular coordinates to polar coordinates and from polar coordinates to rectangular coordinates. Graph equations in the polar coordinate plane.	I can translate equations from rectangular coordinates to polar coordinates using the Pythagorean Theorem and trigonometric ratios. I can convert from polar coordinates to rectangular coordinates using trigonometric ratios. I can graph equations in the polar coordinate plane.	Rectangular coordinates Polar coordinates Polar coordinate plane Pythagorean Theorem	Explain why the rectangular and polar forms of a given complex number represent the same number. (MA.PC.PCN.2)