4-2 Day 1 Notes: Sigma & Area (limit process) Name ____________________

Calculus AB

An extension of indefinite integrations is the definite integral. One of its major applications is finding the area of regions bounded by curves (or combination of curves and lines) that cannot be found using familiar geometric formulas. Definite integration is defined using limits and the sums of many terms. Sigma Notation is used as shorthand to write the sum of a sequence (of a very large number) of terms.

$$\sum_{k=m}^{n}(a_k) = a_m + a_{(m+1)} + a_{(m+2)} + \ldots\ldots + a_{(n-1)} + a_n$$

We can also utilize sigma notation with functions:

$$\sum_{k=m}^{n}(f(k)) = f(m) + f(m+1) + f(m+2) + \ldots\ldots + f(n-1) + f(n)$$

Example 1) Evaluate the following:

a) $\sum_{k=1}^{4}(k) =$

b) $\sum_{i=1}^{n}(i) =$

c) $\sum_{j=0}^{5}(2^j) =$

d) $\sum_{k=1}^{4}(2) =$

e) $\sum_{i=1}^{4}\left(\frac{i-1}{i^2+3}\right) =$

Some Properties of Summation

$\sum_{i=1}^{n}(ka_i) = k\sum_{i=1}^{n}(a_i)$ and $\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n}(a_i) \pm \sum_{i=1}^{n}(b_i)$

Note: these are consistent with our differentiation and integration rules!

Commonly Used Summation Formulas

(proofs are in an appendix of your textbook)

a) $\sum_{i=1}^{n}(k) = k\,n$

b) $\sum_{i=1}^{n}(i) = \frac{n(n+1)}{2}$

c) $\sum_{i=1}^{n}(i^2) = \frac{n(n+1)(2n+1)}{6}$

d) $\sum_{i=1}^{n}(i^3) = \frac{n^2(n+1)^2}{4}$

Example 2) $\sum_{i=1}^{15}(2i-3)$

Example 3) $\sum_{i=1}^{10}\left(i\left(i^2-1\right)\right)$

Calculator Method

sum(seq(expression,variable,lower bound, upper bound, increment))

Note: **sum** is found in the List-Math menu and **seq** is found in the List ops menu

Example 4) Evaluate the following sum for n = 10, 100, 1000, and 10000:

$$\sum_{i=1}^{n}\left(\frac{i+1}{n^2}\right)$$

Example 5) Find the limit of s(n) as $n \to \infty$. $s(n) = \frac{64}{n^3} \cdot \frac{(n)(n+1)(2n+1)}{6}$

Example 6) Use the limit process to find the area of the region between the graph of the function and the x-axis over the indicated interval.

$y = x^2 + 1$ [0, 3]