AP Calculus AB Exploration-

Accumulation and the Anti-Derivative

<u>Review of course up to this beautiful moment in time:</u>

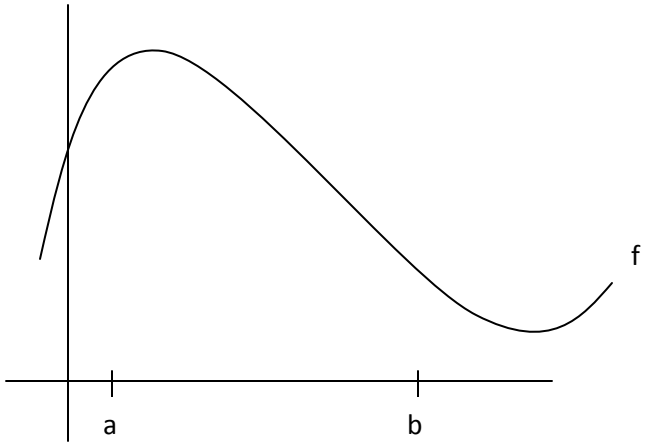
We have covered limits and their use in finding the slope of a tangent line to a curve. We applied this concept of a limit to define and find derivatives of several functions.

Now, we will work "backwards". If the derivative tells us the slope of a function, what would going backwards tell us? If we know that the given function IS the derivative of another function, what would the other function be compared to its derivative?

Derivative is about _____ or _____

Anti-Derivative is about _____ or _____

Consider the following function f:



We know that we could use the derivative to find the slope at any point on the curve, but let's discuss what the anti-derivative or F(x) does.

The anti-derivative accumulates or sums up f. In other words, it finds the area between the graph of f and the x-axis.

Just like with derivatives, there are sets of rules and formulas used to find various anti-derivatives. However, many functions are very complex and finding anti-derivatives can be very challenging and sometimes impossible.

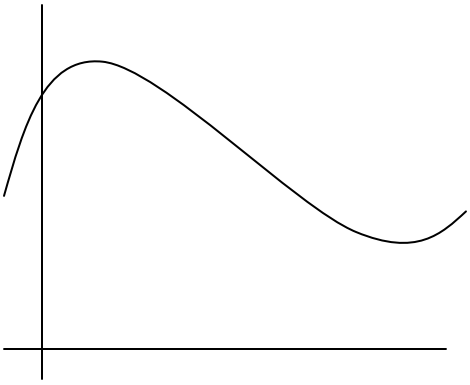
Let's discuss a way to ESTIMATE the area under the curve f from a to b:

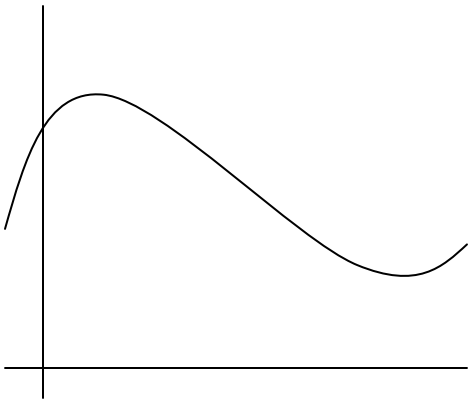
Now, let's formalize this a bit…

We can approximate the area under a curve by "building" rectangles .

We can make our approximation more accurate by:

Left-Handed vs. Right-Handed rectangles:





A left handed rectangle:

A right handed rectangle:

Finding an approximation using rectangles is called a _____ _____.

One other way of building rectangles is to use the _____ to find the height of the rectangle.

This way is sometimes more challenging because we must first find the _____ of each rectangle to plug into f. However, this method often gives a more accurate approximation of the area under the curve.

Class comments/ Notes:

OK, let's try a few together:

Given $f(x) = x^3 + 4$, approximate the area under f from 1 to 5 using four equal sub-intervals with:

a. Right-Riemann Sum

b. Left-Riemann Sum

c. Midpoint-Riemann Sum

Using a table to find a Riemann Sum:

Consider problem #4 from the 2006 AP exam:

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (ft. per sec)	5	14	22	29	35	40	44	47	49

4 b. Using a midpoint Riemann sum with 3 subintervals of equal length, approximate the area of the curve of v(t) from 10 to 70. **Label your answer with correct units and explain what you just found in terms of this particular problem.**

From the 2007 AP exam:

t (min)	0	2	5	7	11	12
r ‘ (t) (ft/min)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical balloon is expanding as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by the function r(t). The table above gives selected values of the rate of change, r ‘ (t), of the radius over the interval $0 \leq t \leq 12$.

1. Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r\,'(t)dt$.

2. Using correct units, explain the meaning of $\int_0^{12} r\,'(t)dt$ in terms of the radius of the balloon.