Indiana Academic Standards

Mathematics: Calculus

I. Introduction

The college and career ready Indiana Academic Standards for Mathematics: Calculus are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The definitions that guided this work were created by the Indiana Education Roundtable, Department of Education, Center for Education & Career innovation, Commission for Higher Education and the Department of Workforce Development. The definition for college and career ready by this group and used throughout this process is as follows: "College-and – career ready means an individual has the knowledge, skills and abilities to succeed in post-secondary education and economically-viable career opportunities." Additionally Public Law 31-2014 [SEA 91] defines college and career readiness educational standards as "the standards that a high school graduate must meet to obtain the requisite knowledge and skill to transition without remediation to post-secondary education or training, and ultimately into a sustainable career."

Standards Process

The Indiana Academic Standards were created through a collaborative process with input from teams of K-12 educators and parents representing school corporations located throughout the state of Indiana; professors of higher education, representing a wide range of Indiana's public and private colleges and universities; and representatives from Indiana businesses and industries. The purpose of the standards process was to design college and career ready standards that would ensure students who complete high school in Indiana are ready for college and careers.

History

Public Law 286 was passed by the Indiana General Assembly in 2013, which created Indiana Code 20-19-2-14.5. The law requires the Indiana State Board of Education to perform a comprehensive review of Indiana's current standards (which were the 2010 Common Core State Standards[1]) and to adopt college and career ready educational standards no later than July 1, 2014.

In the fall of 2013, the Indiana Department of Education established Technical Teams, which were comprised of K-12 educators in English/Language Arts and Mathematics. The Technical Teams were responsible for reviewing the existing Indiana Academic Standards (Common Core State Standards) and providing suggestions for edits and word changes to improve the clarity and progression of the standards. The Department also created Advisory Teams, which were made up of educators from k-12, parents, community members, and higher education institutions across Indiana. The Advisory Teams were responsible for reviewing the work of the Technical Teams and providing additional input.

Evaluation Process

In January of 2014, the Indiana Department of Education, in collaboration with the Indiana State Board of Education, established Evaluation Teams. The Evaluation Teams were responsible for additional layers beyond the work of the Technical and Advisory Teams. The Evaluation Teams were tasked with conducting a comprehensive analysis of several sets of standards, with the goal of identifying the standards that most clearly aligned with the content and skills that Hoosier students would need to know and be able to do in order to be college and career ready.

Membership for the Evaluation Teams was gleaned from individuals who had previously participated on either a Technical Team or an Advisory Team. The Evaluation Team members were selected for their subject matter expertise (in English/Language Arts or Mathematics) and their classroom teaching experience.

[1]

The Evaluation Teams were made up of K-12 educators who represented a wide variety of Indiana school corporations with over 445 years of combined classroom teaching experience, and higher education subject matter experts in English/Language Arts and Mathematics, representing Indiana's public and private institutions of higher education.

The Evaluation Teams met for the first time in February of 2014. The English/Language Arts evaluation teams were given the E/LA Common Core State Standards, as well as Indiana's 2006 E/LA Academic Standards and the standards created by the National Council of Teachers of English. The Mathematics evaluation teams were given the Mathematics Common Core State Standards, as well as Indiana's 2000 Math Academic Standards, Indiana's 2009 Math Academic Standards, and the standards created by the National Council of Teachers of Mathematics.

The panel was instructed to independently evaluate each set of standards, identifying whether the standard was wholly aligned with what a Hoosier student would need to know and be able to do in order to be college and career ready; partially aligned with what a Hoosier student would need to know and be able to do in order to be college and career ready; or not aligned with what a Hoosier student would need to know and be able to do in order to be college and career ready. The results of the evaluation were processed according to a forced consensus requirement—a majority requirement was calculated for each group of standards that was reviewed. Any standard that received a fully aligned rating by the majority of reviewers was marked as fully aligned; any standard that received a not aligned rating by the majority of reviewers was marked as not aligned; and any standard that received a partially aligned rating by the majority, or did not have a majority result, was marked as partially aligned.

Once the evaluations were complete, the results were compiled, and the Evaluation Teams were brought together to conduct a consensus process. The consensus process was blind (meaning that the Evaluation Team members did not know the origin of the standards that they were discussing). Through the consensus process, the Evaluation Teams were asked to select the standards that best and most thoroughly represented what students should know and be able to do in various areas of English/Language Arts and Mathematics in order to be college and career ready. The Evaluation Teams selected the standards that they found to be most appropriate; combined standards to create a more appropriate, rigorous, or clear standard; or, if they determined that gaps existed, wrote standards, or reviewed standards from other states (for example, the English/Language Arts Evaluation Teams reviewed the 2010 draft standards from Massachusetts).

Once the Evaluation Teams had selected the standards (from Common Core State Standards, Indiana Academic, or other states) or had written standards where they found gaps, the list of knowledge and skills identified as necessary for students to be college and career ready was posted for public comment.

Public Comment, Public Hearings, and National Expert Review

The draft college and career ready Indiana Academic Standards were posted for the public to review on February 19, 2014. The public was invited to provide comment through March 12. Over 2000 public comments were received. There were also three public hearings, which were held in southern, central, and northern Indiana, to receive public comment on the draft standards.

The comments from both the online public comment and the public hearings were compiled, reviewed and used to contribute to further iterations of the standards.

In addition, a variety of national experts were contacted to review the draft standards posted on February 19. The results of the reviews were discussed, and portions of the reviews were incorporated into further iterations of the standards.

Reconvening of Evaluation Teams

The Evaluation Teams were reconvened in March of 2014. The teams were tasked with incorporating public comment, and l national expert review to ensure that the draft standards were aligned across grade levels and showed appropriate progression from grade to grade. The Evaluation Teams were also tasked with editing and revising standards for clarity, and addressing any other public comments and national expert review around grade appropriateness, bias, embedded pedagogy, or other factors.

Once the Evaluation Teams completed their reviews, the results were sent to the College and Career Ready (CCR) Panels for final review and approval. The results were also shared with additional national experts, who provided reviews. The results of those reviews were analyzed and synthesized and shared with the CCR Panels.

College and Career Ready (CCR) Panels

The College and Career Ready Panels were created in order to ensure that the standards that Indiana developed were aligned with what colleges, universities, industries, and businesses deem necessary for students to be college and career ready. The CCR Panels were made up of subject matter experts from a variety of Indiana public and private colleges and universities, as well as individuals representing Indiana's businesses and industries.

The CCR Panels were brought together in late March of 2014 to review the draft Indiana Academic Standards that had been reviewed and vetted by the Evaluation Teams in mid-March of 2014. The CCR Panels were tasked with reviewing the standards from 12th grade through kindergarten to ensure that the standards were clear and understandable; aligned across grade levels, showing appropriate progression from grade to grade; and designed to prepare students for college and career readiness. The CCR panels met several times throughout the end of March 2014 and early April 2014 to accomplish this task. At their last meeting, the CCR panel members were asked to sign-off on the draft standards, indicating whether, in their professional opinion, the standards were poised to prepare Hoosier students to be college and career ready.

Indiana Academic Standards

The culmination of the efforts of the Technical Teams, Advisory Teams, Evaluation Teams, and CCR Panels is the college and career ready Indiana Academic Standards that are college and career ready. While many of the standards originated from various sources, including the Common Core State Standards; 2000, 2006, and 2009 Indiana Academic Standards; Massachusetts 2010 Draft English/Language Arts Standards; Virginia Standards of Learning; Nebraska English/Language Arts Standards; the National Council of Teachers of Mathematics; and the National Council of Teachers of English, a number of original standards were also written by members of the Evaluation Teams or CCR Panels.

The process was designed to identify the clearest, most rigorous, and best aligned standards in Mathematics and English/Language Arts to ensure that Hoosier students will graduate meeting the definitions for college and career as defined in Indiana's processes.

<u>What are college and career ready Indiana Academic Standards?</u>

The college and career ready Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The Indiana Academic Standards for English/Language Arts demonstrate what students should know and be able to do in the areas of Reading, Writing, Speaking and Listening, and Media Literacy. The Indiana Academic Standards for Mathematics demonstrate what students should know and be able to do in the areas of K-8 Mathematics; Algebra I, II, and Geometry; and higher-level high school Mathematics courses. The Indiana Academic Standards for Content Area Literacy (History/Social Studies and Science/Technical Subjects) indicate ways in which students should be able to incorporate literacy skills into various content areas at the 6-12 grade levels.

What are the college and career ready Indiana Academic Standards NOT?

1). *The standards are not curriculum.*

While the standards may be used as the basis for curriculum, **the college and career ready *Indiana Academic Standards are not a curriculum*.** Therefore, identifying the sequence of instruction at each grade—what will be taught and for how long—requires concerted effort and attention at the corporation and school levels. While the standards may have examples embedded, and resource materials may include guidelines and suggestions, the standards do not prescribe any particular curriculum. Curriculum is determined locally by a corporation or school and is a prescribed learning plan toward educational goals that includes curricular tools and instructional materials, including textbooks, that are selected by the corporation/school and adopted through the local school board.

2). *The standards are not instructional practices.*

While the standards demonstrate what Hoosier students should know and be able to do in order to be prepared for college and careers, the standards are not instructional practices. The educators and subject matter experts that worked on the standards have taken care to ensure that the standards are free from embedded pedagogy and instructional practices. ***The standards do not define <u>how</u> teachers should teach.*** The standards must be complemented by well-developed, aligned, and appropriate curricular materials, as well as robust and effective instructional best practices.

3). *The standards do not necessarily address students who are far below or far above grade-level.*

The standards are designed to show what the average Hoosier student should know and be able to do in order to be prepared for college and career. However, some students may be far below grade level or in need of special education, and other students may be far above grade level. The standards do not provide differentiation or intervention methods necessary to support and meet the needs of these students. It is up to the district, school, and educators to determine the best and most effective mechanisms of standards delivery for these students.

4). *The standards do not cover all aspects of what is necessary for college and career readiness*

While the standards cover what have been identified as essential skills for Hoosier students to be ready for college and careers, the standards are not—and cannot be—an exhaustive list of what students need in order to be ready for life after high school. Students, especially younger students, require a wide range of physical, social, and emotional supports in order to be prepared for the rigors of each educational progression (elementary grades to middle grades; middle grades to high school; and high school to college or career).

II. Acknowledgements

The college and career ready Indiana Academic Standards could not have been developed without the time, dedication, and expertise of Indiana's K-12 teachers, parents higher education professors, and representatives of Indiana business and industry. Additionally, the members of the public, including parents, community members, policymakers, and educators who took time to provide public comments, whether through the online comment tool or in person at the various public hearings, have played a key role in contributing to the Indiana Academic Standards.

The Indiana Department of Education and Indiana State Board of Education would like to thank Ms. Sujie Shin of the Center on Standards and Assessment Implementation for providing expert facilitation throughout the process and acting in an advisory capacity. The Department and Board would also like to thank the individuals and organizations who provided national expert reviews of the draft standards.

We wish to specially acknowledge the members of the Technical Teams, Advisory Teams, Evaluation Teams, and College and Career Ready Panels who dedicated hundreds of hours to the review, evaluation, synthesis, rewriting, and creation of standards designed to be of the highest quality so that our Hoosier students who are ready for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.
PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

CALCULUS

The Mathematics standards for Calculus are supplemented by the Process Standards for Mathematics.

The Mathematics standards for Calculus are made up of five strands: Limits and Continuity; Differentiation; Applications of Derivatives; Integrals; and Applications of Integrals. The skills listed in each strand demonstrate what students should know and be able to do in Calculus.

CALCULUS	
LIMITS AND CONTINUITY	C.LC.1: Understand the concept of limit and estimate limits from graphs and tables of values.
	C.LC.2: Find limits by substitution.
	C.LC.3: Find limits of sums, differences, products, and quotients.
	C.LC.4: Find limits of rational functions that are undefined at a point.
	C.LC.5: Find limits at infinity.
	C.LC.6: Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. Find special limits $\lim_{x \to 0} \frac{\sin x}{x}$
	C.LC.7: Find one-sided limits.
	C.LC.8: Understand continuity in terms of limits.
	C.LC.9: Decide if a function is continuous at a point.
	C.LC.10: Find the types of discontinuities of a function.
	C.LC.11: Understand and use the Intermediate Value Theorem on a function over a closed interval.
	C.LC.12: Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval.

DIFFERENTIATION	C.D.1: Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change.
	C.D.2: State, understand, and apply the definition of derivative.
	C.D.3: Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions.
	C.D.4: Find the derivatives of sums, products, and quotients.
	C.D.5: Find the derivatives of composite functions, using the chain rule.
	C.D.6: Find the derivatives of implicitly-defined functions.
	C.D.7: Find the derivatives of inverse functions.
	C.D.8: Find second derivatives and derivatives of higher order.
	C.D.9: Find derivatives using logarithmic differentiation.
	C.D.10: Understand and apply the relationship between differentiability and continuity.
	C.D.11: Understand and apply the Mean Value Theorem.
APPLICATION OF DERIVATIVES	C.AD.1: Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents.
	C.AD.2: Find a tangent line to a curve at a point and a local linear approximation.
	C.AD.3: Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f'.
	C.AD.4: Solve real-world and other mathematical problems finding local and absolute maximum and minimum points with and without technology.
	C.AD.5: Analyze real-world problems modeled by curves, including the notions of monotonicity and concavity with and without technology.
	C.AD.6: Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f''. Understand points of inflection as places where concavity changes.

	C.AD.7: Use first and second derivatives to help sketch graphs modeling real-world and other mathematical problems with and without technology. Compare the corresponding characteristics of the graphs of f, f', and f".
	C.AD.8: Use implicit differentiation to find the derivative of an inverse function.
	C.AD.9: Solve optimization real-world problems with and without technology.
	C.AD.10: Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including distance, velocity, and acceleration.
	C.AD.11: Find the velocity and acceleration of a particle moving in a straight line.
	C.AD.12: Model rates of change, including related rates problems.
INTEGRALS	C.I.1: Use rectangle approximations to find approximate values of integrals.
	C.I.2: Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points.
	C.I.3: Interpret a definite integral as a limit of Riemann Sums.
	C.I.4: Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $\int_a^b f'(x)dx = f(b) - f(a)$
	C.I.5: Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.

	C.I.6: Understand and use these properties of definite integrals. $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ $\int_a^b k \cdot f(x)dx = k\int_a^b f(x)dx$ $\int_a^a f(x)dx = 0$ $\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ $\text{If } f(x) \le g(x) \text{ on } [a,b], \text{ then } \int_a^b f(x)dx \le \int_a^b g(x)dx$
	C.I.7: Understand and use integration by substitution (or change of variable) to find values of integrals.
	C.I.8: Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.
APPLICATIONS OF INTEGRALS	C.AI.1: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line.
	C.AI.2: Solve separable differential equations and use them in modeling real-world problems with and without technology.
	C.AI.3: Solve differential equations of the form y' = ky as applied to growth and decay problems.
	C.AI.4: Use definite integrals to find the area between a curve and the x-axis, or between two curves.
	C.AI.5: Use definite integrals to find the average value of a function over a closed interval.
	C.AI.6: Use definite integrals to find the volume of a solid with known cross-sectional area.
	C.AI.7: Apply integration to model and solve (with and without technology) real-world problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral.