This Teacher Resource Guide has been developed to provide supporting materials to help educators successfully implement the Indiana Academic Standards for Probability and Stastics Mathematics – Adopted April 2014. These resources are provided to help you in your work to ensure all students meet the rigorous learning expectations set by the Academic Standards. Use of these resources is optional – teachers should decide which resource will work best in their school for their

The Indiana Department of Education would like to thank Jeremy Eltz for his contributions to this document.

The examples in this document are for illustrative purposes only, to promote a base of clarity and common understanding. Each example illustrates a standard but please note that examples are not intended to limit interpretation or classroom applications of the standards.

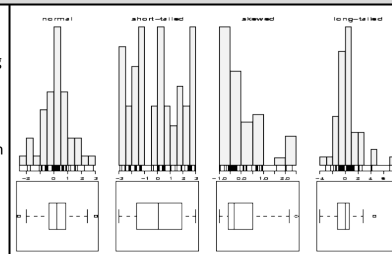
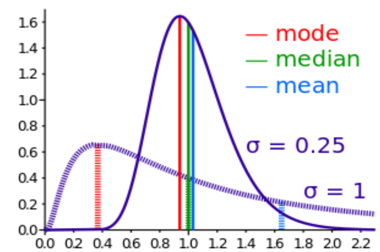
The links compiled and posted in this Resource Guide have been provided by the Department of Education and other sources. The DOE has not attempted to evaluate any posted materials. They are offered as samples for your reference only and are not intended to represent the best or only approach to any particular issue. The DOE does not control or guarantee the accuracy, relevance, timeliness, or completeness of information contained on a linked website; does not endorse the views expressed or services offered by the sponsor of a linked website; and cannot authorize the use of copyrighted materials contained in linked websites. Users must request such authorization from the sponsor of the linked website.

**GOOD WEBSITES FOR MATHEMATICS:**

http://nlvm.usu.edu/en/nav/vlibrary.html
http://www.math.hope.edu/swanson/methods/applets.html
http://learnzillion.com
http://illuminations.nctm.org
https://teacher.desmos.com
http://illustrativemathematics.org
http://www.insidemathematics.org
https://www.khanacademy.org/
https://www.teachingchannel.org/
http://map.mathshell.org/materials/index.php
https://www.istemnetwork.org/index.cfm
http://www.azed.gov/azccrs/mathstandards/

Indiana DEPARTMENT OF EDUCATION

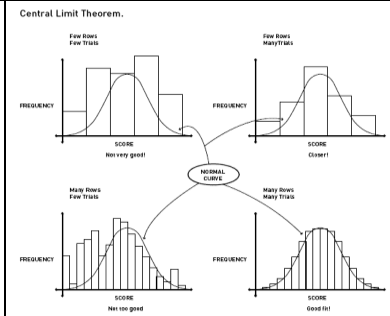
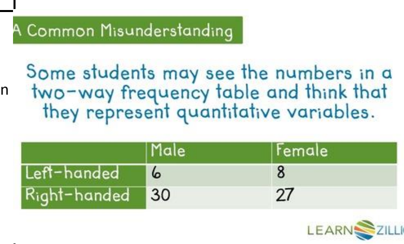
**Indiana Academic Standards for Mathematics – Probability and Statistics**
**Adopted April 2014 – Standards Resource Guide Document**

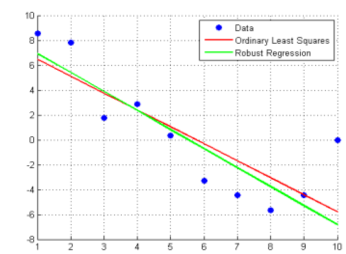
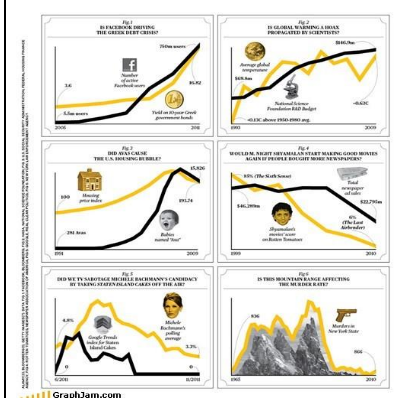
| | Indiana Academic Standard for Mathematics Probability and Statistics – Adopted April 2014 | Highlighted Vocabulary Words from the Standard Defined | Specific Probability and Statistics Example for the Standard | SpecificProbability and Statistics Electronic Resource for the Standard |
|---|---|---|---|---|
| | | **Data Analysis** | | |
| MA.PS.DA.1: | PS.DA.1: Create, compare, and evaluate different graphic displays of the same data, using **histograms, frequency polygons, cumulative frequency distribution functions,** pie charts**, scatterplots, stem-and-leaf plots, and box-and-whisker plots**. Draw these with and without technology. | **Histogram**- graphical representation of the distribution of data<br>**Frequency polygons**- graphical device for understanding the shapes of distributions, serve the same purpose of historgrams<br>**Cumulative frequency distribution functions**-describes the probability that a real-valued random variable X with a given probability distribution will be found to have a value less than or equal to x.<br>**Scatterplots**- type of mathematical diagram using Cartesian coordinates to display values for two variables for a set of data<br>**Stem and Leaf Plot**- A special table where each data value is split into a "leaf" (usually the last digit) and a "stem" (the other digits).<br>**Box-and-whisker plots**-convenient way of graphically depicting groups of numerical data through their quartiles |  | http://www.andrews.edu/~calkins/math/edrm611/edrm02.htm<br><br>http://onlinestatbook.com/2/graphing_distributions/graphing_distributions.pdf |
| MA.PS.DA.2: | PS.DA.2: Compute and use **mean, median, mode, weighted mean, geometric mean, harmonic mean, range, quartiles, variance, and standard deviation**. Use tables and technology to estimate areas under the normal curve. Fit a data set to a **normal distribution** and estimate population percentages. Recognize that there are data sets not normally distributed for which such procedures are inappropriate. | **Mean**-sum of a collection of numbers divided by the number of numbers in the collection<br>**Median**-the numerical value separating the higher half of a data sample, a population, or a probability distribution, from the lower half<br>**Mode**-the value that appears most often in a set of data<br>**Weighted mea**n-similar to an arithmetic mean, where instead of each of the data points contributing equally to the final average, some data points contribute more than others<br>**Geometric mea**n-a type of mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values<br>**Harmonic mean**-one of several kinds of average. Typically, it is appropriate for situations when the average of rates is desired.<br>**Range**-the difference between the largest and smallest values.<br>**Quartiles**-the quartiles of a ranked set of data values are the three points that divide the data set into four equal groups, each group comprising a quarter of the data.<br>**Variance**-measures how far a set of numbers is spread out. A variance of zero indicates that all the values are identical.<br>**Standard deviation**-shows how much variation or dispersion from the average exists<br>**Normal distributio**n-a very commonly occurring |  | http://www.cse.wustl.edu/~jain/cse567-08/ftp/k_12smd.pdf<br><br>http://www.cs.wayne.edu/~hzhang/courses/7290/Lectures/9%20-%20Summarizing%20Measured%20Data.pdf |

**Indiana DEPARTMENT OF EDUCATION**

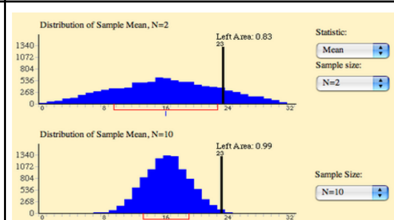
**Indiana Academic Standards for Mathematics – Probability and Statistics**

**Adopted April 2014 – Standards Resource Guide Document**

<table>
<tr><th></th><th>Indiana Academic Standard for Mathematics Probability and Statistics – Adopted April 2014</th><th>Highlighted Vocabulary Words from the Standard Defined</th><th>Specific Probability and Statistics Example for the Standard</th><th>SpecificProbability and Statistics Electronic Resource for the Standard</th></tr>
<tr><td>MA.PS.DA.3:</td><td>PS.DA.3: Understand the central limit theorem and use it to solve problems.</td><td>Central limit theorem-under certain conditions, the sum of many independent identically-distributed random variables, when scaled appropriately, converges in distribution to a standard normal distribution</td><td>
</td><td>https://www.khanacademy.org/math/probability/statistics-inferential/sampling_distribution/v/central-limit-theorem</td></tr>
<tr><td>MA.PS.DA.4:</td><td>PS.DA.4: Understand hypothesis tests of means and differences between means and use them to reach conclusions. Compute and use confidence intervals to make estimates. Construct and interpret margin of error and confidence intervals for population proportions.</td><td>Hypothesis Test- method of statistical inference using data from a scientific study<br>Confidence intervals-a type of interval estimate of a population parameter and is used to indicate the reliability of an estimate<br>Margin of error- a statistic expressing the amount of random sampling error in a survey's results.</td><td><table>
<tr><th></th><th>A</th><th>B</th><th>C</th></tr>
<tr><td>1</td><td>t Test for Differences in Two Means</td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>Hypothesized Difference</td><td>0</td><td></td></tr>
<tr><td>4</td><td>Level of Significance</td><td>0.05</td><td></td></tr>
<tr><td>5</td><td>Population 1 Sample</td><td></td><td></td></tr>
<tr><td>6</td><td>Sample Mean</td><td>51</td><td></td></tr>
<tr><td>7</td><td>Sample Size</td><td>81</td><td></td></tr>
<tr><td>8</td><td>Sample Standard Deviation</td><td>4</td><td></td></tr>
<tr><td>9</td><td>Population 2 Sample</td><td></td><td></td></tr>
<tr><td>10</td><td>Sample Mean</td><td>48</td><td></td></tr>
<tr><td>11</td><td>Sample Size</td><td>64</td><td></td></tr>
<tr><td>12</td><td>Sample Standard Deviation</td><td>3.4641</td><td></td></tr>
<tr><td>13</td><td>Population 1 Sample Degrees of Freedom</td><td>80</td><td></td></tr>
<tr><td>14</td><td>Population 2 Sample Degrees of Freedom</td><td>63</td><td></td></tr>
<tr><td>15</td><td>Total Degrees of Freedom</td><td>143</td><td></td></tr>
<tr><td>16</td><td>Pooled Variance</td><td>14.23775731</td><td></td></tr>
<tr><td>17</td><td>Difference in Sample Means</td><td>3</td><td></td></tr>
<tr><td>18</td><td>t-Test Statistic</td><td>4.753888125</td><td></td></tr>
<tr><td>19</td><td></td><td></td><td></td></tr>
<tr><td>20</td><td>Upper-Tail Test</td><td></td><td></td></tr>
<tr><td>21</td><td>Upper Critical Value</td><td>1.65558049</td><td></td></tr>
<tr><td>22</td><td>p-Value</td><td>2.40729E-06</td><td></td></tr>
<tr><td>23</td><td></td><td></td><td></td></tr>
</table></td><td>https://www.khanacademy.org/math/probability/statistics-inferential/hypothesis-testing-two-samples/v/hypothesis-test-for-difference-of-means<br><br>http://www.ltcconline.net/greenl/courses/201/hyptest/diff.htm</td></tr>
<tr><td>MA.PS.DA.5:</td><td>PS.DA.5: Recognize how linear transformations of univariate data affect shape, center, and spread.</td><td>Linear Transformation- a mapping V ↦ W between two modules (including vector spaces) that preserves the operations of addition and scalar multiplication<br>Univariate data-data involving a single variable</td><td>http://www.youtube.com/watch?v=9gcNxUcVJck</td><td>http://stattrek.com/statistics/charts/data-patterns.aspx</td></tr>
<tr><td>MA.PS.DA.6:</td><td>PS.DA.6: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.</td><td>Two-way frequency tables-a visual representation of the possible relationships between two sets of categorical data<br>Conditional Probabilities-measures the probability of an event given that another event has occurred.</td><td>
</td><td>http://mathbitsnotebook.com/Algebra1/StatisticsReg/ST2TwoWayTable.html</td></tr>
<tr><td>MA.PS.DA.7:</td><td>PS.DA.7: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.</td><td></td><td></td><td>http://www.cpalms.org/Public/PreviewStandard/Preview/5651</td></tr>
</table>

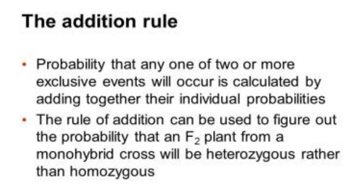
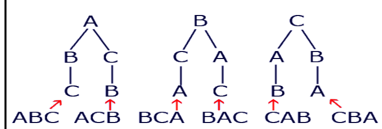
| | Indiana Academic Standard for Mathematics Probability and Statistics – Adopted April 2014 | Highlighted Vocabulary Words from the Standard Defined | Specific Probability and Statistics Example for the Standard | SpecificProbability and Statistics Electronic Resource for the Standard |
|---|---|---|---|---|
| MA.PS.DA.8: | PS.DA.8: Understand the meaning of measurement data and **categorical data**, of univariate and bivariate data, and of the term variable. | **Categorical data**-data that can take on one of a limited, and usually fixed, number of possible values | | |
| MA.PS.DA.9: | PS.DA.9: Understand statistics and use **sampling distributions** as a process for making inferences about **population parameters** based on a random sample from that population. | **Sampling Distribution**-or finite-sample distribution is the probability distribution of a given statistic based on a random sample<br>**Population parameter**s-numerical characteristic of a population | | |
| MA.PS.DA.10 | PS.DA.10: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | http://people.wku.edu/david.neal/statistics/projects/project1.html | https://www.checkmarket.com/2013/02/how-to-estimate-your-population-and-survey-sample-size/ |
| MA.PS.DA.11 | PS.DA.11: Find linear models by using **median fit** and **least squares regression** methods to make predictions. Decide which among several linear models gives a better fit. Interpret the slope and intercept in terms of the original context. Informally assess the fit of a function by plotting and analyzing residuals. | **Median fit**- the line of best fit, used to indicate a relation or trend in data sets<br>**Least squares regression**-mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offsets) of the points from the curve. |   | http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=4&ved=0CDsQFjAD&url=http%3A%2F%2Fwww3.wabash.edu%2FeconExcel%2FLMSOrigin%2FLMSIntro.doc&ei=r1WeU9vOOY2cyAT-4IGICg&usg=AFQjCNGjvKSoPxlWqb243tBz6CfQRcFwlg&bvm=bv.68911936,d.aWw |
| MA.PS.DA.12 | PS.DA.12: Evaluate reports based on data by considering the source of the data, the design of the study, the way the data are analyzed and displayed, and whether the report confuses **correlation** with **causation**. Distinguish between correlation and causation. | **Correlation**-a relation between two variables, does not imply causation<br>**Causation**-causal relationship between conduct and result |   | http://www.abs.gov.au/websitedbs/a3121120.nsf/home/statistical+language+-+correlation+and+causation |

**Indiana Academic Standards for Mathematics – Probability and Statistics**
**Adopted April 2014 – Standards Resource Guide Document**

| | Indiana Academic Standard for Mathematics Probability and Statistics – Adopted April 2014 | Highlighted Vocabulary Words from the Standard Defined | Specific Probability and Statistics Example for the Standard | SpecificProbability and Statistics Electronic Resource for the Standard |
|---|---|---|---|---|
| | | **Experimental Design** | | |
| MA.PS.ED.1: | PS.ED.1: Formulate questions that can be addressed with data. Collect, organize, and display relevant data to answer the questions formulated. | | http://www.txprofdev.org/apps/datadecisions/node/47.html | http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&ved=0CCYQFjAB&url=http%3A%2F%2Fwww4.uwm.edu%2FOrg%2Fmmp%2FPPTs-Yr9%2FAsking_Stats_Questions_and_Collecting_Data_Powerpoint_S3-2_09-20-2011.ppt&ei=w1aeU9WZAoWmyATU4IKYDg&usg=AFQjCNFVMckhSU_COr-wB-M0Hb6cvO3NAQ&bvm=bv.68911936,d.aWw |
| MA.PS.ED.2: | PS.ED.2: Use **election theory** techniques to analyze election data. Use **weighted voting** techniques to decide voting power within a group. | **Election theory**-the mathematical treatment of the process by which democratic societies or groups resolve the many and conflicting opinions of the members of the group into a single choice of the group.<br>**Weighted voting**- systems based on the idea that not all voters are equal | http://www.cpalms.org/Public/PreviewStandard/Preview/235 | https://www.avon-schools.org/cms/lib02/IN01001885/Centricity/Domain/3488/FA%20Ch%202%20Notes%20weighted%20voting%20INKED%20SOLUTIONS%20pdf.pdf |
| MA.PS.ED.3: | PS.ED.3: Construct simulated sampling distributions of sample proportions and use sampling distributions to identify which proportions are likely to be found in a sample of a given size. | **Sampling distributions**-finite-sample distribution is the probability distribution of a given statistic based on a random sample | Sampling Distribution<br>Variable / Parameter / Statistic / Center / Spread / Shape<br>Categorical (example: left-handed or not) / p = population proportion / $\hat{p}$ = sample proportion / p / $\sqrt{\frac{p(1-p)}{n}}$ / Normal IF np ≥ 10 and n(1 - p) ≥ 10<br>Quantitative (example: age) / μ = population mean, σ = population standard deviation / $\bar{x}$ = sample mean / μ / $\frac{\sigma}{\sqrt{n}}$ / **When will the distribution of sample means be approximately normal?** | http://onlinestatbook.com/2/sampling_distributions/samp_dist_p.html |
| MA.PS.ED.4: | PS.ED.4: Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions. | |  | http://www.psychstat.missouristate.edu/introbook/sbk19m.htm<br><br>http://www.learner.org/courses/againstallodds/pdfs/AgainstAllOdds_StudentGuide_Set3.pdf |
| MA.PS.ED.5: | PS.ED.5: Model and solve real-world problems using the geometric distribution or waiting-time distribution, with or without technology. | **Geometric distribution**-either of two discrete probability distributions:<br>The probability distribution of the number X of Bernoulli trials needed to get one success, supported on the set { 1, 2, 3, ...}<br>The probability distribution of the number Y = X − 1 of failures before the first success, supported on the set { 0, 1, 2, 3, ... }<br>**Waiting-time distribution**-the probability distribution that describes the time between events in a Poisson process, i.e. a process in which events occur continuously and independently at a constant average rate | http://www.youtube.com/watch?v=fyOdJ34iMpU | http://stats.stackexchange.com/questions/60975/comparison-of-waiting-times-to-geometric-distribution<br><br>http://higheredbcs.wiley.com/legacy/college/watkins/0470458518/addonline/thegeometricdistribution.pdf |

| | Indiana Academic Standard for Mathematics Probability and Statistics – Adopted April 2014 | Highlighted Vocabulary Words from the Standard Defined | Specific Probability and Statistics Example for the Standard | SpecificProbability and Statistics Electronic Resource for the Standard |
|---|---|---|---|---|
| MA.PS.ED.6: | PS.ED.6: Model and solve real-world problems involving patterns using recursion and iteration, growth and decay, and compound interest. | | If a recursive method is called with a base case, the method returns a result. If a method is called with a more complex problem, the method divides the problem into two or more conceptual pieces: a piece that the method knows how to do and a slightly smaller version of the original problem. Because this new problem looks like the original problem, the method launches a recursive call to work on the smaller problem.<br>For recursion to terminate, each time the recursion method calls itself with a slightly simpler version of the original problem, the sequence of smaller and smaller problems must converge on the base case. When the method recognizes the base case, the result is returned to the previous method call and a sequence of returns ensures all the way up the line until the original call of the method eventually returns the final result.<br>Both iteration and recursion are based on a control structure: Iteration uses a repetition structure; recursion uses a selection structure.<br>Both iteration and recursion involve repetition: Iteration explicitly uses a repetition structure; recursion achieves repetition through repeated method calls.<br>Iteration and recursion each involve a termination test: Iteration terminates when the loop-continuation condition fails; recursion terminates when a base case is recognized.<br>Iteration and recursion can occur infinitely: An infinite | http://www.cs.cornell.edu/info/courses/spring-98/cs211/lecturenotes/07-recursion.pdf<br><br>http://infolab.stanford.edu/~ullman/focs/ch02.pdf |
| MA.PS.ED.7: | PS.ED.7: Understand and apply basic ideas related to the design, analysis, and interpretation of surveys and sampling, such as background information, random sampling, causality and **bias**. | **Bias**-prejudice in favor of or against one thing | | http://home.ubalt.edu/ntsbarsh/stat-data/surveys.htm |
| MA.PS.ED.8: | PS.ED.8: Understand how basic statistical techniques are used to monitor process characteristics in the workplace. | | | http://www.collegeboard.com/prod_downloads/yes/4297_MODULE_05.pdf |
| MA.PS.ED.9: | PS.ED.9: Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each. | | | http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&ved=0CCYQFjAB&url=http%3A%2F%2Fwww.amstat.org%2Feducation%2Fstew%2Fpdfs%2FChocolicious.docx&ei=eFueU5G8KoatyATfmoDIBg&usg=AFQjCNGs6H9vzcqdVRgtSLnC7_Z6rllAjw&bvm=bv.68911936,d.aWw |

Indiana Academic Standards for Mathematics – Probability and Statistics
Adopted April 2014 – Standards Resource Guide Document

| | Indiana Academic Standard for Mathematics<br>Probability and Statistics – Adopted April 2014 | Highlighted Vocabulary Words<br>from the Standard Defined | Specific Probability and Statistics Example for the Standard | SpecificProbability and Statistics Electronic Resource for the Standard |
|---|---|---|---|---|
| | | **Probability** | | |
| MA.PS.P.1: | PS.P.1: Understand and use the addition rule to calculate probabilities for mutually exclusive and nonmutually exclusive events. | **Addition rule**-When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event.<br>P(A or B) = P(A) + P(B)<br><br>**Mutually Exclusive**-2 events that cannot occur at the same time |  | http://www.mathgoodies.com/lessons/vol6/addition_rules.html<br><br>http://statistics.about.com/od/Formulas/a/Addition-Rules-In-Probability.htm |
| MA.PS.P.2: | PS.P.2: Understand and use the multiplication rule to calculate probabilities for independent and dependent events. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | **Multiplication rule**- a mthod for finding the probability that both of two events occur | | |
| MA.PS.P.3: | PS.P.3: Understand the **multiplication counting principle**, **permutations, and combinations**; use them to solve real-world problems. Use simulations with and without technology to solve counting and probability problems. | **Multiplication counting principle**: If there are a ways for one activity to occur, and b ways for a second activity to occur, then there are a • b ways for both to occur.<br>**Permutations**- rearranging, members of a set into a particular sequence or order<br>**Combinations**- a way of selecting members from a grouping, such that the order of selection does not matter |  | http://dmc122011.delmar.edu/math/pjohnson/Webpage/businessmath/notes/9.2.pdf<br><br>http://www.mhhe.com/math/precalc/barnettpc5/graphics/barnett05pcfg/ch10/others/bpc5_ch10-05.pdf |
| MA.PS.P.4: | PS.P.4: Calculate the probabilities of **complementary events**. | **Complementary event**s-those events where the probability of one event precludes the happening of the other event | The probability of getting a white ball from a bag of balls is $\frac{1}{4}$<br>$P(\text{ball is not white}) = 1 - \frac{1}{4} = \frac{3}{4}$ | http://www.mathsisfun.com/data/probability-complement.html |
| MA.PS.P.5: | PS.P.5: Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | | Find the mean of the following probability distribution?<br>X P(X)<br>1 0.20<br>2 0.10<br>3 0.35<br>4 0.05<br>5 0.30<br>Mean of a discrete probability distribution (as this one) is given by<br>Sum x*P(x) = 1*0.2 + 2*0.1 + 3*0.35 + 4*0.05 + 5*0.3 = 3.15 | http://www.statisticshowto.com/how-to-find-the-mean-of-the-probability-distribution-or-binomial-distribution/ |
| MA.PS.P.6: | PS.P.6: Analyze decisions and strategies using probability concepts. Analyze probabilities to interpret odds and risk of events. | | | http://www.vaoutcomes.org/downloads/probability_and_odds_ratio.pdf |

| | Indiana Academic Standard for Mathematics Probability and Statistics – Adopted April 2014 | Highlighted Vocabulary Words from the Standard Defined | Specific Probability and Statistics Example for the Standard | SpecificProbability and Statistics Electronic Resource for the Standard |
|---|---|---|---|---|
| MA.PS.P.7: | PS.P.7: Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. | | A probability distribution is a table or an equation that links each outcome of a statistical experiment with its probability of occurrence. Consider the coin flip experiment described above. The table below, which associates each outcome with its probability, is an example of a probability distribution.<br>Number of heads Probability<br>0 0.25<br>1 0.50<br>2 0.25 | http://www.stats.gla.ac.uk/steps/glossary/probability_distributions.html<br><br>https://www.khanacademy.org/math/probability/random-variables-topic/random_variables_prob_dist/v/random-variables |
| MA.PS.P.8: | PS.P.8: Develop a **probability distribution** for a random variable defined for a sample space in which theoretical probabilities can be calculated; Compute and interpret the expected value of random variables. | **Probability distribution**- assigns a probability to each measurable subset of the possible outcomes of a random experiment, survey, or procedure of statistical inference | | http://www.stats.gla.ac.uk/steps/glossary/probability_distributions.html<br><br>https://www.khanacademy.org/math/probability/random-variables-topic/random_variables_prob_dist/v/random-variables |
| MA.PS.P.9: | PS.P.9: Derive the **binomial theorem** by combinatorics. Use combinatorial reasoning to solve problems. | **Binomial theorem**-describes the algebraic expansion of powers of a binomial, "Pascal's triangle" | | https://www.khanacademy.org/math/algebra2/polynomial_and_rational/binomial_theorem/v/binomial-theorem--part-3 |
| MA.PS.P.10: | PS.P.10: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as **unions, intersections**, or complements of other events. | **Union**-set of all distinct elements in the collection<br>**Intersection**-the set of elements common to different sets | http://sites.stat.psu.edu/~jiali/course/stat416/notes/chap1.pdf | http://www.math.uiuc.edu/~kkirkpat/SampleSpace.pdf |