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DEPARTMENT OF EDUCATION

Working Together for Student Success



Indiana Academic Standards Mathematics: Quantitative Reasoning



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Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards have been developed through the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
<p>PS.1: Make sense of problems and persevere in solving them.</p>	<p>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p>
<p>PS.2: Reason abstractly and quantitatively.</p>	<p>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p>

<p>PS.3: Construct viable arguments and critique the reasoning of others.</p>	<p>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</p>
<p>PS.4: Model with mathematics.</p>	<p>Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>
<p>PS.5: Use appropriate tools strategically.</p>	<p>Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as</p>

	<p>digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.</p>
<p>PS.6: Attend to precision.</p>	<p>Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.</p>
<p>PS.7: Look for and make use of structure.</p>	<p>Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.</p>
<p>PS.8: Look for and express regularity in repeated reasoning.</p>	<p>Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.</p>

MATHEMATICS: Quantitative Reasoning

Numeracy	
QR.N.1	Represent quantities in equivalent forms (fractions, decimals, and percentages) to investigate and describe quantitative relationships and solve real-world problems in a variety of contexts. Compare the size of numbers in different forms arising in authentic real-world contexts, such as growth expressed as a fraction versus as a percentage. Interpret the meaning of numbers in different forms, such as the meaning of a fraction or the meaning of a percentage greater than 100 and its validity in a given context. Recognize incorrect or deceptive uses of fractions, decimals, or percentages.
QR.N.2	Solve problems involving calculations with percentages and interpret the results, such as calculating percentage rates or differentiating between a discount of 30% and two consecutive discounts of 15%. Calculate relative change and explain how it differs from absolute change. Recognize incorrect or deceptive uses of percentages.
QR.N.3	Interpret numbers in different forms in terms of authentic contexts to solve real-world problems, such as interpreting a growth rate less than 1%. Compare and precisely communicate with numbers in different forms (including words, fractions, decimals, standard notation, and scientific notation), such as comparing relative and absolute changes in quantities.
QR.N.4	Compare magnitudes of numbers in context, such as the population of the US compared to the population of the world. Perform such comparisons when numbers are in different forms (including words, fractions, decimals, standard notation, and scientific notation).
QR.N.5	Perform accurate and efficient calculations using large and small numbers in different forms, to an appropriate precision, with and without technology. Include calculations in context, such as ratios representing water use per capita for a large population.
QR.N.6	Use estimation skills, and know why, how, and when to estimate results. Identify and use numeric benchmarks for estimating calculations (e.g., using 25% as an estimate for 23%). Identify and use contextual benchmarks for comparison to other numbers (e.g., using the US population as a benchmark to evaluate reasonableness of statistical claims or giving context to numbers). Check for reasonableness using both types of benchmarks.



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QR.N.7

Use dimensional analysis to convert between units of measurements and to solve problems involving multiple units of measurement, such as converting between currencies, calculating the cost of gasoline to drive a given car a given distance, or calculating dosages of medicine.

Ratio and Proportional Reasoning

QR.P.1	Solve real-life problems requiring interpretation and comparison of complex numeric summaries which extend beyond simple measures of center, such as problems requiring interpreting and/or comparing weighted averages, indices, coding, and ranking. Evaluate claims based on complex numeric summaries.
QR.P.2	Understand and communicate percentages as rates per 100, and identify uses and misuses of percentages related to a proper understanding of the base in real-world and mathematical problems.
QR.P.3	Solve real-life problems requiring interpretation and comparison of various representations of ratios, (i.e. fractions, decimals, rate, and percentages), such as problems that involve non-standard ratios (e.g., media and risk reporting) or part-to-part versus part-to-whole ratios taken from meaningful context.
QR.P.4	Analyze growth and decay using absolute and relative change and make comparisons using absolute and relative difference.
QR.P.5	Distinguish between proportional and non-proportional situations, and, when appropriate, apply proportional reasoning, such as when solving for an unknown quantity in proportional situations; solving real-life problems requiring conversion of units using dimensional analysis; or applying scale factors to perform indirect measurements (e.g., maps, blueprints, concentrations, dosages, and densities). Recognize when proportional techniques do not apply.
QR.P.6	Determine the constant of proportionality in proportional situations (both real-life and mathematical), leading to a symbolic model for the situation (i.e. an equation based upon a rate of change, $y = kx$).

Modeling	
QR.M.1	Analyze and critique mathematical models and be able to describe their limitations, including distinguishing between correlation and causation and determine whether interpolation and/or extrapolation are appropriate.
QR.M.2	Use models, including models created with spreadsheets or other tools, to estimate solutions to contextual questions, such as functional models to estimate future population or spreadsheets to model financial applications (e.g. credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios). Identify patterns and identify how changing parameters affect the results.
QR.M.3	Choose and create, with and without technology, linear, exponential, logistic, or periodic models and curves of best fit for bivariate data sets. Use the models to answer questions and draw conclusions or make decisions, addressing limitations and long-term ramifications of chosen models when appropriate. Recognize when a change in model is needed.
QR.M.4	Analyze real-world problem situations and use variables to construct and solve equations involving one or more unknown or variable quantities to answer questions about the situations, such as creating spreadsheet formulas to calculate prices based on percentage mark-up or solving formulas for specified values. Demonstrate understanding of the meaning of a solution. Identify when there is insufficient information given to solve a problem.
QR.M.5	Apply geometric concepts to model situations and solve problems such as those arising in art, architecture, and other fields.
QR.M.6	The student uses a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems, such as in scheduling or routing situations that can be modeled using different methods, e.g., vertex-edge graphs using critical paths, Euler paths, or minimal spanning trees.

Probabilistic Reasoning to Assess Risk

QR.P.1	Determine the nature and number of elements in a finite sample space to model the outcomes of real-world events using counting techniques, and build the sample space by making lists, tables, or tree diagrams.
QR.P.2	Determine the number of ways an event may occur using the Fundamental Counting Principle.
QR.P.3	Evaluate the validity of claims based on empirical, theoretical, and subjective probabilities. Draw conclusions or make decisions related to risk, pay-off, expected value, and false negatives/positives in various probabilistic contexts.
QR.P.4	Use data displays and models, such as two-way tables, tree diagrams, Venn diagrams, and area models, to determine probabilities (including conditional probabilities) and use these probabilities to make informed decisions.

Statistics	
QR.S.1	Analyze statistical information from studies, surveys, and polls (including when reported in condensed form or using summary statistics) to make informed judgments as to the validity of claims or conclusions, such as when interpreting and comparing the results of polls using margin of error.
QR.S.2	Identify limitations, strengths, or lack of information in studies, including data collection methods (e.g. sampling, experimental, observational) and possible sources of bias, and identify errors or misuses of statistics to justify particular conclusions.
QR.S.3	Create (with and without technology) and use visual displays of real world data, such as charts, tables and graphs.
QR.S.4	Interpret and analyze visual representations of data, and describe strengths, limitations, and fallacies of various graphical displays.
QR.S.5	Read, interpret, and make decisions about data summarized numerically using measures of center and spread, in tables, and in graphical displays (line graphs, bar graphs, scatterplots, and histograms), e.g., explain why the mean may not represent a typical salary; explain the difference between bar graphs and histograms; critique a graphical display by recognizing that the choice of scale can distort information.
QR.S.6	Summarize, represent, and interpret data sets on a single count or measurement variable using plots and statistics appropriate to the shape of the data distribution to represent it.
QR.S.7	Compare center, shape, and spread of two or more data sets and interpret the differences in context.
QR.S.8	Use properties of distributions, including uniform and normal distributions, to analyze data and answer questions.
QR.S.9	Recognize when data are normally distributed and use the mean and standard deviation of the data to fit it to a normal distribution.



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