Avon High School **Name** Key

AP Calculus AB

UNIT 4 EXAM – Extrema, Special Theorems, First and Second Derivative Tests

Period _____ **Score** ______ **/ 36** ______ **/ 100**

Part I. Multiple Choice: Graphing Calculator Permitted

1. The first derivative of the function f is given by $f'(x)=\frac{\cos^2 x}{x}-\frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

a. Seven **b.** Five **c.** Four **d.** Three **e.** One

2. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

a. Four **b.** Three **c.** Two **d.** One **e.** None

3. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$?

a. 2 only **b.** −1 only **c.** −1 and 2 only **d.** −1 and 0 only **e.** −1, 0, and 2 only

4. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

a. $\frac{2\pi}{3}$ **b.** $\frac{3\pi}{4}$ **c.** $\frac{5\pi}{6}$ **d.** π **e.** $\frac{3\pi}{2}$

5. State a(some) reason(s) why the Rolle's Theorem does not apply to the function $f(x)=\frac{2}{(x+1.5)^2}$ on the interval [-3, 0].

a. f is not defined at x = -3 and x = 0 **b.** $f(-3)\neq f(0)$ **c.** f is not continuous at x = -1.5

d. Both b and c. **e.** None of these.

Part II. Free Response: Graphing Calculator Permitted

6. For some key values of x, the values of a continuous function, $f(x)$, $f'(x)$, and $f''(x)$ are given in the table below.

x	-8	-6	-4	-2	0	2	4	6
$f(x)$	6	2	0	-4	-6	0	2	0
$f'(x)$	-3	0	-3	-1	undefined	2	0	-3
$f''(x)$	2	0	-3	0	undefined	0	-2	-4

a. Identify all x-values where f has a relative maximum. Justify specifically.

$\boxed{x=4}$ $\boxed{f'(4)=0 \ \& \ f''(4)<0}$

or $f'(x)$ changes from pos. to neg. at $x=4$

b. Identify all x-values where f has a relative minimum. Justify specifically.

$\boxed{x=0}$ $\boxed{f'(0) \text{ DNE } \& \ f'(x) \text{ changes from neg to pos at } x=0}$

c. Identify the x-value where f has a point of inflection. Justify specifically.

$\boxed{x=-6}$ $\boxed{f''(6)=0 \ \& \ f''(x) \text{ changes signs at } x=-6}$

d. What is the equation of the tangent to the curve $y=f(x)$ at $x=-2$.

Point: $(-2,-4)$ $\boxed{y+4=-(x+2)}$

$m=-1$

or $y=-x-6$

e. Does Rolle's Theorem apply for the function $f(x)$ if it were to be defined on the interval [-6, 4]? Justify your answer.

$\boxed{\text{No}}$

$\boxed{f(x) \text{ is not differentiable at } x=0}$

No Calculator --- No Calculator --- No Calculator --- No Calculator --- No Calculator --- No Calculator

Part III. Multiple Choice: No Calculator is permitted.

+ − +

7. Let $f''(x) = 3x^2 - 4$ and let $f(x)$ have first derivative critical numbers at -2, 0, and 2. Use the **second derivative test** to determine which critical number(s), if any, give a relative minimum.

a. -2 **b.** 0 **c.** 2 **d.** 0 and -2 **e.** -2 and 2

8. Given $f(x) = 6 - \frac{10}{x}$, find all c in the interval (2, 5) such that $f'(c) = \frac{f(5) - f(2)}{5 - 2}$.

a. $\pm\sqrt{10}$ **b.** $\pm\sqrt{15}$ **c.** $\sqrt{10}$ **d.** $\sqrt{15}$ **e.** None of these

$f'(x) = \frac{10}{x^2}$ $f'(c) = \frac{4-1}{5-2} = 1$

$\frac{10}{x^2} = 1$

$10 = x^2$

$x = \pm\sqrt{10}$

9. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflections for the graph of f?

a. 1 **b.** 0 **c.** -1 **d.** 0 and 1 **e.** -1, 0 and 1

$f'(x) = 15x^4 - 20x^3$

$f''(x) = 60x^3 - 60x^2$

$0 = 60x^2(x-1)$

$x = 0, 1$

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
$f''(x)$	−	−	+

10. Let f be a function defined and continuous on the closed interval $[a, b]$. If f has a relative maximum at c and $a < c < b$, which of the following must be true?

I. $f'(c)$ must exist.

✓**II.** If $f'(c)$ exists, then $f'(c) = 0$.

✓**III.** If $f''(c)$ exists, then $f''(c) \le 0$.

a. II only **b.** III only **c.** I and II only **d.** I and III only **e.** II and III only

No Calculator --- No Calculator --- No Calculator --- No Calculator --- No Calculator --- No Calculator

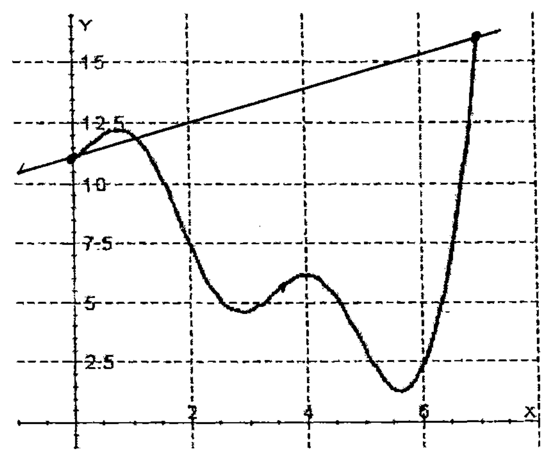
11. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

✓ **I.** f is continuous at $x = 0$.
× **II.** f is differentiable at $x = 0$.
✓ **III.** f has an absolute minimum at $x = 0$.

a. I only **b.** II only **c.** III only **d.** I and III only **e.** I, II and III only

12. Use the graph of f on the right to estimate the value of c that justifies the Mean value Theorem for the interval [0, 7].

a. 7 **b.** 5.5 **c.** 4.3
d. 3.7 **e.** 3



13. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

$c = \pm 2$

$(-\infty,-2)$ $(-2,2)$ $(2,\infty)$

$f'(x)$ $-$ $+$ $-$

min at $x=-2$ max at $x=2$

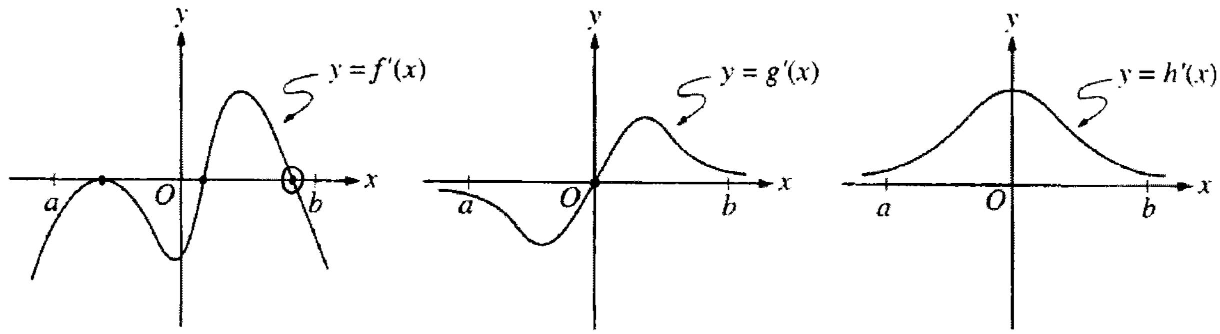
a. f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
b. f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
c. f has relative maxima at $x = -2$ and $x = 2$.
d. f has relative minima at $x = -2$ and $x = 2$.
e. It cannot be determined if f has any relative extrema.

14. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

a. $x < -1$ *or* $x > 3$ **b.** $x < -3$ *or* $x > 1$ **c.** $-1 < x < 1$

d. $-3 < x < 1$ **e.** All real numbers

$f'(x) = 3x^2 + 6x - 9$
$0 = 3(x^2 + 2x - 3)$
$0 = (x+3)(x-1)$
$x = -3, 1$
$(-\infty, -3)$ $(-3,1)$ $(1,\infty)$
$f'(x)$ $+$ $-$ $+$



15. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g or h have a relative maximum on the open interval $a < x < b$?

a. f only **b.** g only **c.** h only **d.** f and g only **e.** f, g, and h

16. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

a. -2 **b.** 0 **c.** 1 **d.** 2 **e.** 4

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

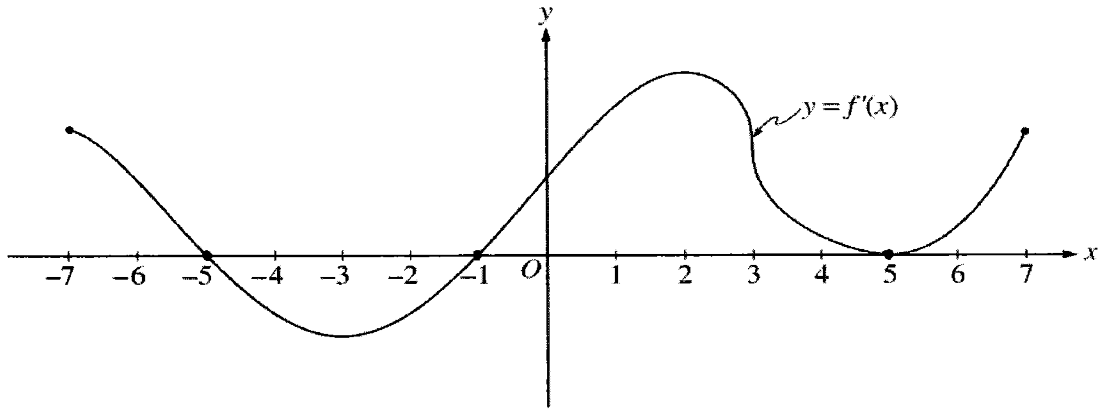
$x = 0, 2$

x	f(x)
0	12
2	8
-2	-8
4	28

No Calculator --- No Calculator --- No Calculator --- No Calculator --- No Calculator --- No Calculator

Part IV. Free Response: No Calculator is permitted.

17. The figure below shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and , $x = 5$, and a vertical tangent at $x = 3$.



a. Find all values of x, for $-7 \le x \le 7$, at which f attains a relative minimum. Justify your answer.

$x = -1$

$f'(x)$ changes from neg. to pos. at $x=-1$

b. Find all values of x, for $-7 \le x \le 7$, at which f attains a relative maximum. Justify your answer.

$x = -5$

$f'(x)$ changes from pos. to neg at $x=-5$

c. Find all intervals for which $f''(x) < 0$.

$(-7,-3)$ \& $(2,5)$

d. At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum?

$x = 7$